Resistor Tolerance Estimation vs. Resistor Value Standard Deviation
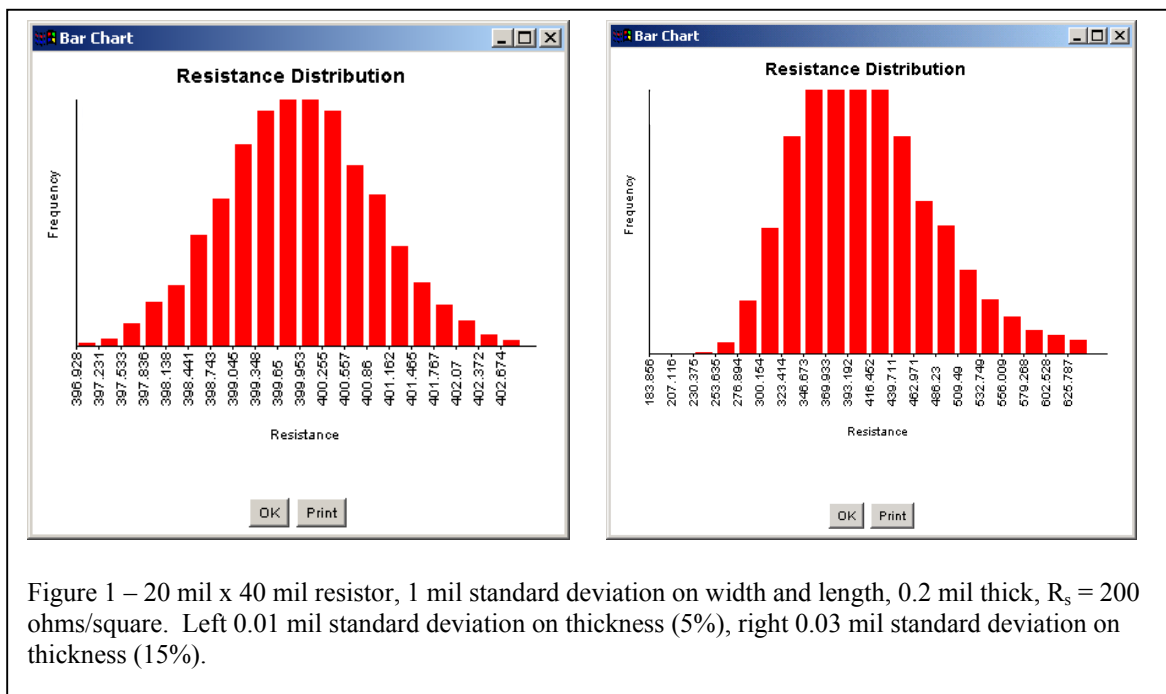
I performed the follow experiment:

1) A simple Monte Carlo analysis of the following calculation of resistor value,
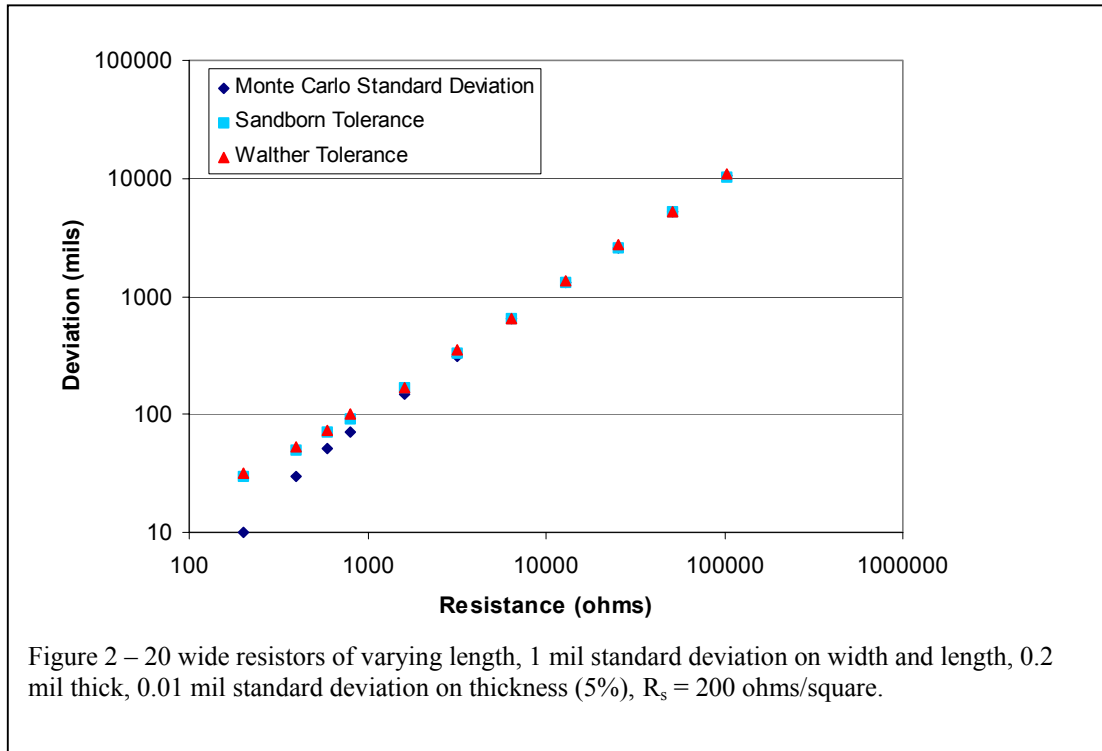
$$R = R_s \frac{T_{mean}}{T} \frac{L}{W} \tag{1}$$

I used normal distributions for the L, W, and T (the length, width, and thickness) of the resistor (note, we need not treat variations in R_s and T separately, one or the other will do). I ran 5,000,000 samples and obtained the resulting histograms of resistance results (examples are shown in Figure 1).
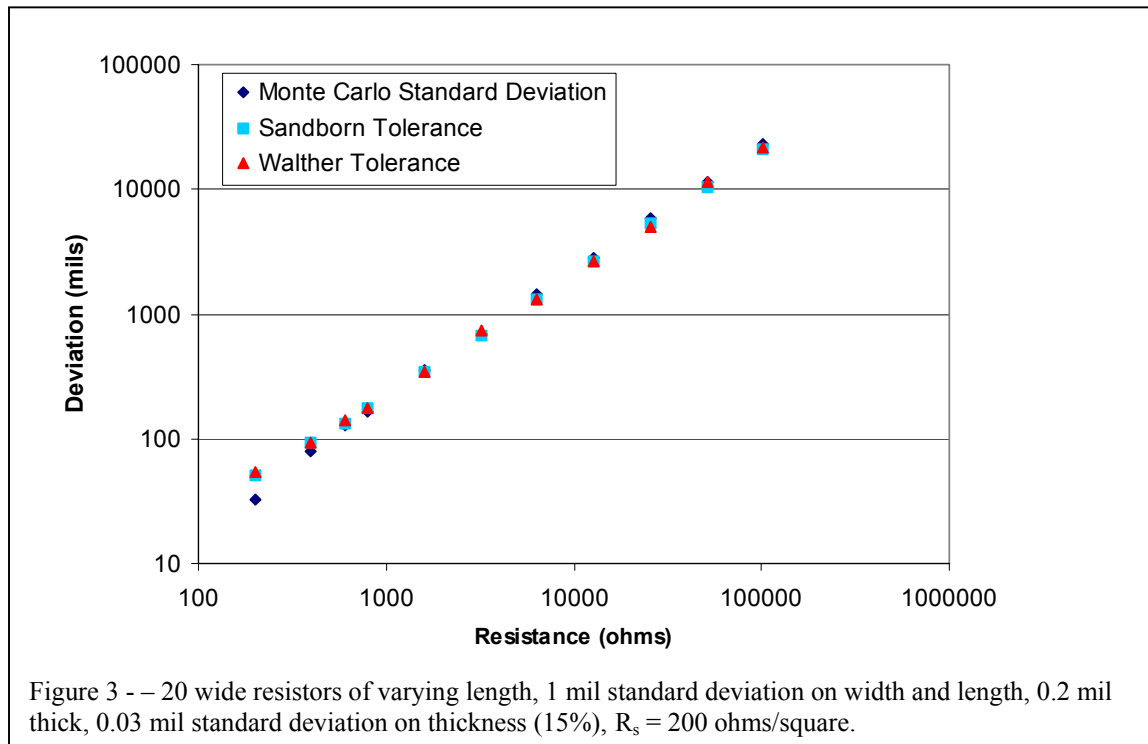

Figure 1 – 20 mil x 40 mil resistor, 1 mil standard deviation on width and length, 0.2 mil thick, R_s = 200 ohms/square. Left 0.01 mil standard deviation on thickness (5%), right 0.03 mil standard deviation on thickness (15%).

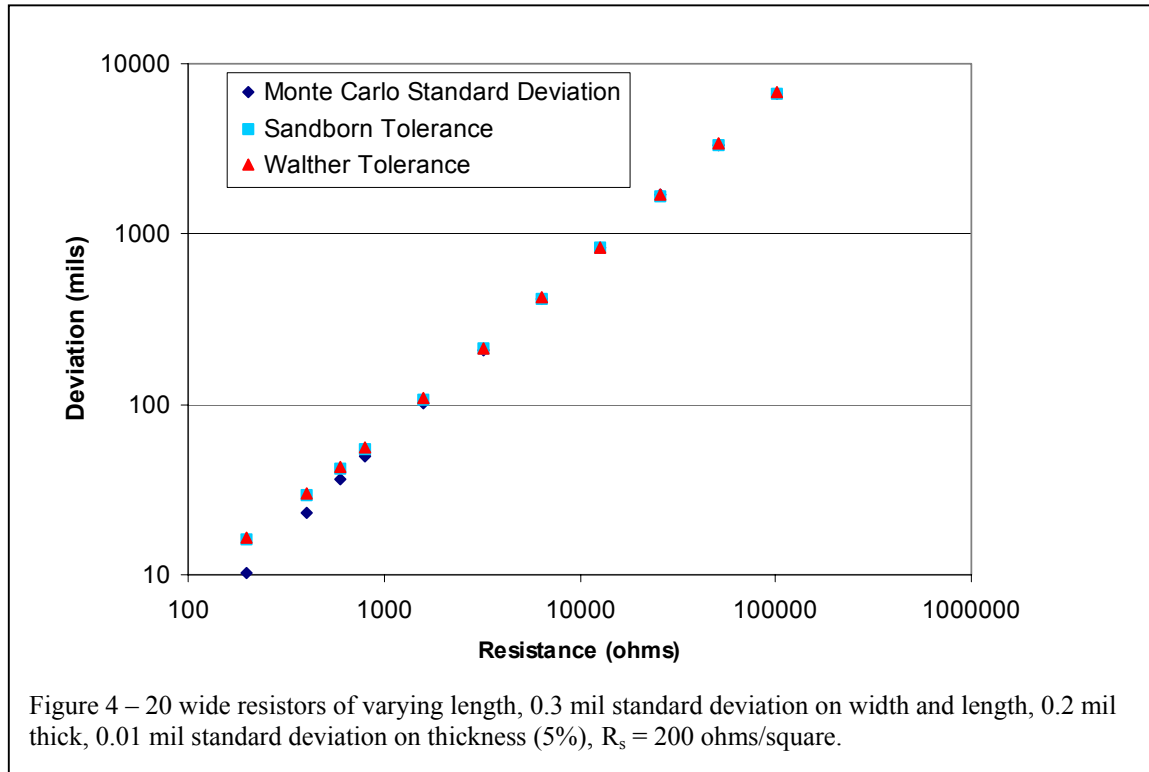Note, the distributions in Figure 1 tend to skew to the right as the actual distributions do.

2) Next, I compared the standard deviation in resistance value (obtained from the Monte Carlo analysis above) with the tolerances predicted by the Walther and Sandborn models. One thing to be clear on here, we expect the tolerance models to be conservative, i.e., they should predict a variation that is greater than (or equal to) the actual variation from the Monte Carlo model. When I plot the predicted deviations against the resistance values (assuming a 20 mil wide resistor with varying length) I get Figure 2.

Figure 2 – 20 wide resistors of varying length, 1 mil standard deviation on width and length, 0.2 mil thick, 0.01 mil standard deviation on thickness (5%), R_s = 200 ohms/square.
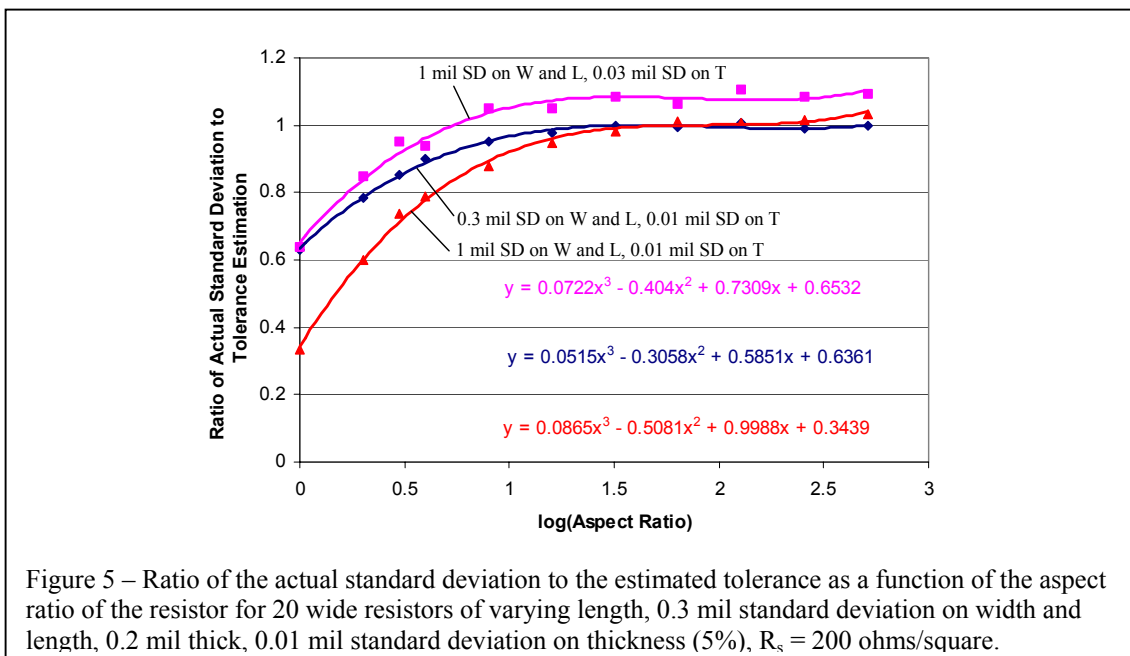
As can be seen in Figure 2, the tolerance calculations match the actual standard deviation prediction for large resistors and over-predict the standard deviation for smaller resistors. The relation shown in Figure 2 is a function of the standard deviations on the width, length, and thickness. For example, the same data with a thickness standard deviation of 0.03 mils is shown in Figure 3.

Figure 3 - – 20 wide resistors of varying length, 1 mil standard deviation on width and length, 0.2 mil thick, 0.03 mil standard deviation on thickness (15%), R_s = 200 ohms/square.

If the standard deviation on the width and length is varied for example, Figure 4 is obtained. Comparing Figures 2 and 4 shows that as the standard deviation in the width and length decreases, the tolerance estimations predict the standard deviation in the resistance value more accurately (i.e., for smaller resistors).

Figure 4 – 20 wide resistors of varying length, 0.3 mil standard deviation on width and length, 0.2 mil thick, 0.01 mil standard deviation on thickness (5%), R_s = 200 ohms/square.

Note, you can take this one more step and derive the relationship between the simple tolerance estimations and the actual standard deviation in the resistance value, Figure 5.

Figure 5 – Ratio of the actual standard deviation to the estimated tolerance as a function of the aspect ratio of the resistor for 20 wide resistors of varying length, 0.3 mil standard deviation on width and length, 0.2 mil thick, 0.01 mil standard deviation on thickness (5%), R_s = 200 ohms/square.

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Another finding – the two tolerance models (Walther and Sandborn) are virtually identical. The Sandborn version of the model looks like it fits the actual standard deviations slightly better for large resistance values.