Evaluating the End of Maintenance Dates for Electronic Assemblies Composed of Obsolete Parts

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Long-term support of legacy electronic systems is challenging due to mismatches between the system support life and the procurement lives of the systems' constituent components. Legacy electronic systems that are used in safety, mission and infrastructure critical applications that must be supported for 20+ years are threatened with Diminishing Manufacturing Sources and Material Shortages (DMSMS)-type obsolescence, and their effective system support lives may be governed by existing non-replenishable inventories of spare parts. This paper describes the development of the End of Maintenance (EOM) model, which uses a stochastic discrete-event simulation that follows the life history of the population of parts in a system using time-to-failure distributions and other forecasted demands. The model determines the support life of the system based on existing inventories of spare parts and cards, and optionally harvesting parts from existing cards to extend the support life of the system. The model includes: part inventory degradation, periodic inventory inspections, and design refresh planning for selected cards.

A case study using a real legacy system comprised of 117,000 instances of 70 unique cards and 4.5 million unique parts is presented. The case study was used to evaluate the support life of a system with various future failure assumptions, including with and without the use of part harvesting. The case study also includes sensitivity analyses for selected design refreshes to maximize potential system life-cycle capabilities, and optional design refresh planning required to sustain the system to a specific date.

Keywords: sustainment, COTS, legacy systems, demand forecasting, EOR, EOM, harvesting, DMSMS, obsolescence, design refresh

1 Introduction

The long-term sustainment of electronic systems in safety, mission and infrastructure critical applications is a challenging task for system supporters. These types of systems must often be supported for 20+ years, have relatively low production volumes and are highly qualified and certified making system changes prohibitively expensive or impossible. Example systems include avionics, industrial controls, power plant control systems, military systems, etc. The support challenge for these systems includes the reliability of fielded components, required system availability, and supply chain and inventory management, all while trying to minimize system life-cycle costs. Due to the nature of the electronic part market, safety, mission and infrastructure critical applications usually depend on the same Commercial Off The Shelf (COTS) components that high-volume consumer applications (e.g., cell phones, tablet computers) depend on.

The required use of COTS electronic components creates difficulties for many applications – notably they bind their users to volatile market trends where technology continuously evolves [1,2]. This rapid technological evolution quickly introduces "new and improved" components; however, this evolutionary process creates ripple effects that hinder legacy[1] electronic system supporters [3]. Specifically, the emergence of electronic part obsolescence (DMSMS-type obsolescence) whereby a component is no longer procurable from its original manufacturer is a problem that plagues many legacy system supporters [3]. DMSMS-type obsolescence is a significant contributor to systems becoming "sustainment-dominated;" sustainment-dominated systems are systems whose long-term sustainment (life-cycle) costs exceed their original procurement costs [4]. Sustainment-dominated systems become legacy systems when it becomes too expensive to replace them with newer systems.

The focus for legacy electronic system supporters becomes minimizing system life-cycle cost while maximizing system support. Part obsolescence often limits a system's support life. DMSMS-type obsolescence is typically resolved through a variety of reactive mitigation approaches. Reactive approaches, although not a solution to the obsolescence problem, provide the supporter with ways to manage the problem tactically. Reactive mitigation approaches include the use of alternate or substitute parts, aftermarket sources, lifetime buys, thermal uprating of parts, and emulated parts [5,6]. Proactive management includes part criticality analysis, critical spare part stocking, and maintenance planning [7]. Strategic solutions that involve planning design refreshes based on forecasted part obsolescence also exist, e.g., [8,9], however, due to their potentially large expense and management/budget constraints, design refreshes may not be practical or possible for many systems. The strategies focused on in this paper are those that use existing inventories of parts and reclamation to extend system support life based upon currently owned excess components and fielded legacy assemblies.

One of the most widely used reactive approaches is a lifetime buy. A lifetime buy (also known as a life-of-type buy) means purchasing enough parts when the part becomes obsolete to satisfy all foreseeable future demand. As a result of lifetime buys, legacy system supporters are often left with potentially non-replenishable inventories of parts to support the system into the future. For many applications that have

[1] Legacy systems are defined as fielded and operational systems for which no new system production or replacement is planned.

indefinite end of support dates,[2] the question asked by supporters is "on what date will the system become unsupported?"

1.1 End of Maintenance Definition. A population of systems is composed of a set of system instances, e.g., a system instance could be one ground radar installation at one airport that is a member of a population of systems comprising all ground radar instances at all airports. Systems are composed of a set of "cards". A card is an assembly composed of one or more parts, e.g., a card could be a printed circuit board, and the parts could be electronic chips.

The Federal Aviation Administration (FAA) defines *End of Maintenance* (EOM) as "the moment a site requisition cannot be replenished. This stage change begins with the depletion of limited depot and site spares quantities, followed by service degradation (i.e., loss of redundancy) and ultimately loss of system operations" [10]. The last portion of the FAA's definition (loss of system operations) is what the model described in this paper defines as the EOM date for the system. In our model, the EOM date is defined as "the earliest date that all available inventories fail to support the demand for one or more specific parts resulting in the loss of system operation." An EOM event is defined for a population of systems but is caused by a specific part on a specific card.

Additionally, the FAA [10] defines *End of Repair* (EOR) as "when hardware product support is no longer available by any means or is cost-prohibitive." In this paper, the EOR date is defined as "the date that the last repair or manufacturing action associated with a part can be successfully performed." EOR events are part specific if all cards can draw from all inventories (no inventory segregation) and EOR events may be part and card specific if specific cards can only draw parts from specific inventories.

1.2 Demand Forecasting. Demand forecasting is an important piece of inventory management and plays a significant role in electronic systems sustainment modeling (including EOM modeling). Demand is

[2] This is a very common situation for military and civil infrastructure systems for which there is effectively no end of support date, i.e., "ageless systems" (or end of support dates are regularly extended). For example, consider air traffic control systems, while replacement of these types of aging systems has been proposed, the date on which the actual replacement will occur is unclear and depends more on economic and political pressures than system realities. In some cases the type of analysis described in this paper is a necessary step in making a business case to replace legacy systems.

composed of part needs to support future manufacturing of new system instances and part needs to support maintaining existing systems in the field.

Several different approaches to demand forecasting have appeared in the literature. First, a significant body of literature is devoted to "inventory optimization". Inventory optimization models determine the stock size that minimizes the system cost. Most inventory optimization models are extensions of the basic Economic Order Quantity (EOQ) model [11], which minimizes the sum of the part holding and the order costs. Inventory optimization models exist for both single and multi-echelon cases, for deterministic and stochastic demand, repairable and non-repairable systems, and a wide variety of other variations. Several reviews of inventory optimization models exist, including [12,13]. While many inventory optimization models focus exclusively as production demand, there are also inventory optimization models that specifically treat the "spare parts" problem, e.g., [14-17]. In some cases [14] these approaches incorporate reliability models for determining spare part demand into the inventory optimization process. Fundamentally, inventory optimization models address how many parts should be held in inventory.

The demand forecasting problem that needs to be addressed to evaluate EOMs is not an inventory optimization problem, i.e., in the EOM problem, the inventories of parts already exist and cost minimization is not the objective. The objective is to accurately forecast part demands based on predicted or historical part-specific failure data and then use it to assess the length of the possible system support period. A common problem among mission-critical systems is that their system lives are unknown until well into the sustainment period and the system must continue to be sustained until otherwise determined. Note, the EOM model does not model scheduled retirements.

General sparing analysis depends on demand forecasting for maintenance of existing systems. The irregular nature of spares demand (i.e., some time periods have zero demand and others do not), makes the number of spares difficult to predict. Croston's method [18] is the best known model for intermittent demand. Croston's method involves separate exponential smoothing forecasts on the size of a demand and the time period between demands. Croston's model, and numerous extensions to it [17,19], provide only point forecasts and are not based on a stochastic model. Deterministic methods only provide point forecasts and cannot produce forecast distributions and demand prediction intervals. A stochastic process allows for a multitude of probable and possible solutions based upon associated uncertainties, allowing for system complexities to be explored. Stochastic models incorporate the inherent randomness associated with spare parts demands, meaning that demands for a part arise only when that part actually fails. The derivation for stochastic analysis of demand forecasting for service parts and approximate closed-form solutions have been proposed for constant part failure rates and constant part discard rates. For example, the stochastic model developed in [19] determines part demands for given periods of time based on periodic product sales and failure information. Eppen and Martin [20] investigate two cases considering stochastic demand size and lead times in a given period where the parameters of the distributions are known and unknown - when the parameters are unknown, they use a simple exponential smoothing model to generate estimates of demand in each period. Sandmann and Bober [21] fit real failure data to suitable stochastic models using Maximum Likelihood Estimation (MLE) and use it to estimate propose stock control policies.

Unfortunately, the stochastic models mentioned in the previous paragraph only forecast part demands considering one part type during their analysis. Real systems contain hundreds (maybe thousands) of unique parts where each part could be characterized by a different failure distribution and by different distribution parameters.[3] The objective is not only to forecast multiple part demands stochastically and simultaneously, but to track the system support life information (part failures and spares inventory depletion) over time as the forecasted part demands occur as 'events' that change the system. The ability to forecast individual part demands is included in the example stochastic models mentioned above, but the manner and organization of how this process is carried out (i.e., the sequence of failures in time) is also of importance and it is not accommodated in [19] and [20].

In order to capture the sequence of part failures in time for large numbers of unique parts, the model described in this paper uses a discrete event simulator to follow the life history of a population of parts and cards.[4] We simulate an entire system support lifetime through the stochastic process of forecasting many part demands (a timeline that progresses through part demands) simultaneously while retaining the ability to track at any instant during the simulation what particular event is taking place (exactly what part fails and when the failure occurs). Note, electronic parts (piece-parts, e.g., chips) are assumed to be non-

[3] It is also possible that the same part appearing in different locations in the same system has different reliability characteristics.

[4] Alternative stochastic models that could be considered include Markov chain models and Petri nets.

repairable, so each part failure results in a part replacement event. The next section describes the model, which is followed by a demonstration of the model using a detailed case study.

2 End of Repair/End of Maintenance (EOR/EOM) Model

The discrete event simulator described in this paper models the process of inventory depletion through system operation and support for legacy systems and tracks the end of repair and end of maintenance dates for the system. The electronic systems hierarchy assumed in the model includes parts and cards defined in Section 1.1. In real systems, maintenance activities are unique based on the component that has caused the system to fail, the detail and complexity of varying maintenance lengths and procedures, depending on the component are outside the scope of this work and are not considered in the EOM model. As the model is a discrete-event simulation, it captures only an event, which encapsulates the entire logistics footprint (from failure to logistics, maintenance, test/check, repair/replace, and reintroduction into an operational state) and therefore, all piece-parts are "maintained" equally and instantaneously at the event in this model.

The model tracks information regarding individual parts from introduction to the field, through failure and replacement, and possibly subsequent failures and replacements through the system support life until EOM (or end of support) occurs. An example process flow for parts within an electronic system is shown in Fig. 1.

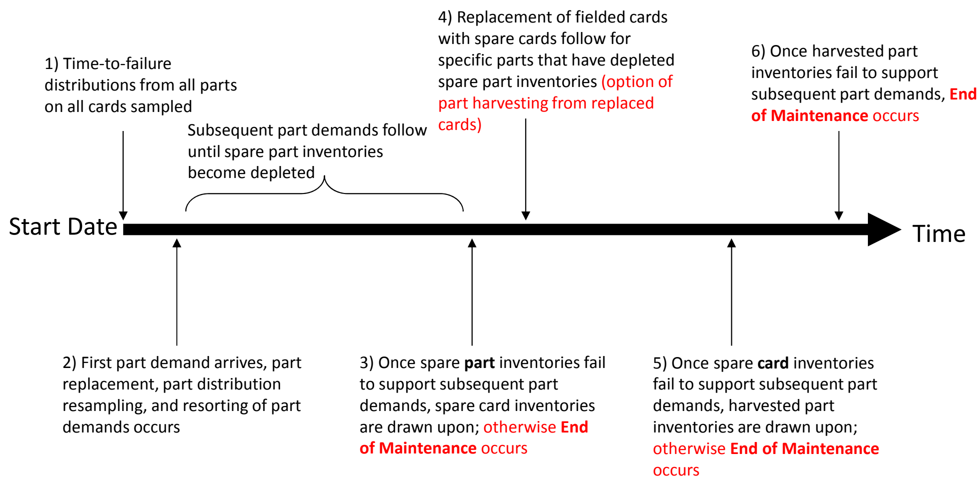


Fig. 1 EOR/EOM part failure process flow (single life history).

The model starts by sampling time-to-failure distributions of individual parts that are located on cards within the system.[5] After all of the part demand dates are sampled within the system, the demand dates are sorted from earliest to latest on a part-by-part basis.[6] The model then determines the earliest part demand that occurs in the system, moves to its date, and performs a change to the system (this type of change is dependent on the type of event that occurs). After the change has been applied, the second earliest part demand is found, the model moves to its date, and the process continues. The model continues until a part demand cannot be fulfilled by the existing inventories (i.e., inventory stock-out).

The simulation begins at the analysis start date and the simulation time progresses until an End of Maintenance event occurs (where a part demand cannot be fulfilled by existing inventories that previously sustained it) --this constitutes a single simulated life history of the entire system. In order to provide an accurate representation of the system support life considering part demand uncertainties, multiple system life histories are tracked (typically 500), which result in probability distributions of End of Maintenance dates and the identification of the parts/cards (and their associated likelihoods) that cause system support loss.

[5] Note, at the start of the analysis the system has already been fielded for a substantial length of time and the parts currently in the systems are not "good as new." Therefore, sampling from the time-to-failure distributions begins prior to the analysis start date.
[6] A part is defined as a component specific to a particular card and retains its own card-specific unique properties (time-to-failure distribution and quantity). Each instance of a part on a card is treated individually (represented by its own forecasted part demand based on sampling from the part's time-to-failure distribution).

2.1 Part Demand Sampling. Only parts that are obsolete at the analysis start date are considered in demand forecasting within the model; all the non-obsolete parts are assumed to be continuously procurable as needed. The number of forecasted part demands is proportional to the total number of fielded part instances within the entire system because each part is considered unique within the model and tracked independently using discrete forecasted demands. The part demands are sampled using Monte Carlo from probability distributions representing the reliability of the parts. Often organizations supporting legacy systems are uncertain of the failure distributions the parts within their system exhibit, but they may have maintenance records of observed part failures. If a sufficient number of failures have been observed the data can be fit to create a time-to-failure distribution (an example appears in the test case included in this paper). When only a few failures exist and no other information is known, a uniform distribution can be generated with lower bound *a* and upper bound *b* for a particular part,

$$a = (D_{FF} - D_S)O_P \tag{1}$$

$$b = \frac{(D_A - D_S)O_P - a}{\left(\frac{N_f}{N_T}\right)} + a \tag{2}$$

where,

D_{FF} = date (in years) of the first failure
D_S = date (in years) the part was fielded
D_A = date (in years) of the start of analysis
O_P = operational hours per year
N_f = number of failures to date
N_T = total number of fielded parts within entire system.

The upper boundary of the distribution, *b*, is dependent upon the ratio of failures to date (between the date the part was fielded and the start of the analysis) divided by the total number of fielded parts. When the ratio, $\frac{N_f}{N_T}$ equals 1, the upper bound becomes the difference between the start of analysis and the date the parts were fielded. Likewise, as $\frac{N_f}{N_T}$ approaches 0 (no failures observed), the upper limit of the distribution approaches ∞. When $\frac{N_f}{N_T}$ approaches ∞ (a large number of failures relative to the population of fielded parts), the width of the distribution approaches zero, i.e., $a = b$.

For cases when obsolete parts in the system have no prior failure history two situations are considered: 1) best-case – assumes the part never fails, and 2) worst case - assumes immediate first failure just prior to the start of analysis.

2.2 Determining EOR/EOM Information. End of repair and end of maintenance events are recorded within every simulated life history of the system. The information associated with each of these events is also recorded and analyzed at the end of the simulation. The model has the ability to track multiple EOR/EOM events within a given system life history. The EOR and EOM events are referred to as "ordered" events based on their chronological occurrences throughout a given system support lifetime. The calculated EOR/EOM information that is analyzed is based on their chronological order of occurrences.

The *i*th-ordered mean EOM time (organized by order of occurrence within a single life history) for a particular part-card combination is given by,

$$\overline{M}_i = \sum_{j=1}^{k} \frac{M_{ij}}{N_{ij}} + D_A \tag{3}$$

where,

$\overline{M}_i$ = *i*th-ordered mean EOM time
M_{ij} = *i*th-ordered EOM time in the *j*th life history
N_{ij} = number of occurrences as an *i*th-ordered EOM in the *j*th life history - either 1 (occurs) or 0 (does not occur)
k = number of life histories simulated.

The corresponding EOM probability for the particular part-card combination causing the *i*th-ordered EOM is given by,

$$P_i = \frac{\sum_{j=1}^{k} N_{ij}}{k} \tag{4}$$

where, P_i = *i*th-ordered EOM probability.

The mean End of Repair times for particular part-card combinations and their associated probabilities are analyzed in the same manner based on the last successful repair action.

The EOM event information can also be organized at the card-level (by card) rather than the system-level (order of occurrence). The associated means and probabilities are generated to provide probability distributions of end of maintenance dates on a card-basis rather than an ordering basis. The card-level EOM information tracked was organized for first-ordered events associated with each card. Therefore, the mean EOM time and corresponding probability for a particular part-card combination concerning its first EOM event is given by equations (3) and (4) with i = 1, respectively.

The EOR/EOM calculated information is the same for card-level and system-level analysis, except that the organizational structures of both analyses are different. The system-level analysis partitions events by order of their occurrences, while the card-level analysis partitions first ordered events by particular cards.

2.3 Spare Card Inventories, Throwaway, Part Harvesting, and Degradation. The model described so far draws from inventories as parts are needed, until the inventories are depleted. The model also includes inventories of spare cards that can be accessed once the part inventories are depleted; potentially further extending the system support life. In the event that a part demand cannot be satisfied for a particular card that has available spare cards to draw from, the existing card is thrown away and replaced with one of the available spare cards. Spare cards, and their constituent parts, are assumed to be "good-as-new". The action of throwaway and replacement of an existing card means the removal and re-sampling of all of its part demands for all of its parts from their corresponding reliability distributions.

There is an intermediate step that may optionally be performed before an existing card is thrown away—existing parts that have not failed may be harvested prior to the card's disposal (i.e., this is an obsolescence mitigation strategy commonly known as reclamation). Another purpose for this study was to investigate the benefit gained from the implementation of harvesting policy to harvest usable parts off of previously discarded cards to be used later when new spares run out.

The action of part harvesting removes non-failed parts from the discarded card and places them in a separate inventory of harvested parts. When inventories of spare parts and spare cards are depleted, this third inventory is then accessed and used until there are no more replacements available. Generally, the physical removal of the part from the assembly will degrade or damage the part further (e.g., additional stress from mishandling, bent pins, etc.).

This condition is accounted for in the model by calculating the fraction of remaining useful life for the *i*th harvested part from the *j*th card, L_{ij}, given by,

$$L_{ij} = H_i \frac{FD_i - t_H}{FD_i - t_i} \tag{5}$$

where,

H_i = preserved life fraction of *i*th part incurred from the physical action of harvesting (1 = no damage to the part from the harvesting process, 0 = all remaining life removed by the harvesting process)

FD_i = *i*th part forecasted demand date

t_H = simulation time when the harvesting action occurs

t_i = simulation time when *i*th part was fielded (t_i =0 for initial fielded parts)

The numerator in the fraction of equation (5) represents the remaining part life as the difference between the forecasted part's demand date and the simulation time at the time harvesting occurs. The denominator represents the part's time-to-failure from the instant the part was fielded. The remaining part life must be preserved as a fraction rather than a time-to-failure because it is likely the part will be used towards replacement on a different card where there may be discrepancies between the parts' reliabilities (i.e., they may be represented by different probability distributions or distribution parameters). The remaining life fraction is then used to adjust future forecasted part demands used during part replacements when all other existing inventories are depleted.

The adjusted forecasted demand of the *i*th part from the *m*th card (*m* may equal *j*, i.e., the harvested part may be replaced on the same type of card), AFD_{im}, is then given by,

$$AFD_{im} = L_{ij} FD_{im} \tag{6}$$

where,

FD_{im} = *i*th part forecasted demand date from the *m*th card

Another possible issue impacting the number of parts available is degradation within inventories (an example could be finite shelf life for a part in inventory). In discrete-event simulations, events can be described as anything that causes a change to the state of the system. The model interprets inventory degradation as an event that can occur at any time. The model distinguishes between part failures and part degradation as two different types of events, but executes their processes in a similar manner (see Fig. 2).

First, the model generates the initial number of demands associated with each type of event included in the analysis. Next, the model cycles through the possible types of events that can occur and finds the earliest date associated with each event type. Some events may never occur during a system life history (i.e., no part degradation) in which case those events are ignored during the evaluation. The model determines the earliest date among all event types and jumps ahead to that date, implements the appropriate changes to the system (dependent on the type of event), removes the date and event that just occurred, and resamples the distribution corresponding to the part (or inventory) that caused the current event. Afterwards, the concurrent event evaluation continues until the EOM is reached.

Due to the nature of discrete-event simulation, the part inventory degradation event can be emulated through assignments of probability distributions representing the likelihood of a part degrading in the inventory in a given time period. The forecasted degradation date for the *i*th part from the *k*th inventory FDD_{ik} is given by,

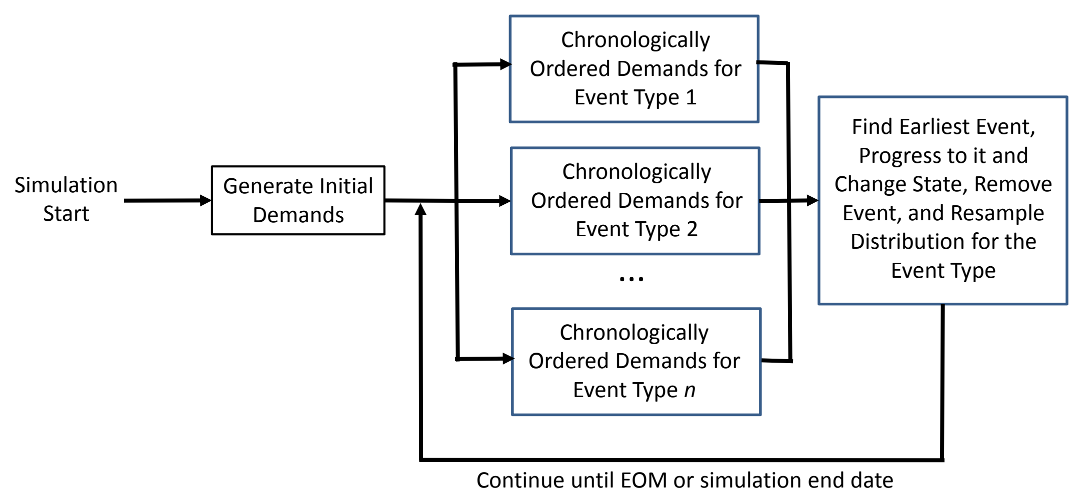


Fig. 2 Discrete-event simulation flow for multiple events.

$$FDD_{ik} = DD_{ik} + t \tag{7}$$

where,

DD_{ik} = ith part forecasted degradation demand date from the kth inventory

t = current simulation time (starting at $t = 0$).

Upon reaching DD_{ik}, the model identifies that a single part has degraded (become unusable) within the inventory. The degradation distribution of the part is then resampled and the next forecasted degradation date is calculated and the process continues until either the inventory of spares runs out or the EOM date is reached.

The process described above identifies the moment that a part has degraded; however, the degraded part is not removed from the inventory at this time. The degraded parts are discarded either at the next inspection event associated with the inventory or after an attempted use of the degraded part is made (i.e., the probability of pulling a degraded part or 'good' part from inventory). For example, the probability at any given time that a degraded part is chosen from the inventory is represented by,

$$\frac{G_{ik}}{NP_{ik}} \tag{8}$$

where,

G_{ik} = number of ith parts considered degraded from the kth inventory

NP_{ik} = number of ith parts remaining from the kth inventory.

If $\frac{G_{ik}}{NP_{ik}}$ equals 0, then none of the ith parts from the kth inventory are considered to be degraded. Conversely, when $\frac{G_{ik}}{NP_{ik}}$ equals 1, then all of the ith parts from the kth inventory are considered to be degraded. Randomly pulling a part from inventory can be represented by randomly choosing a number between 0 and 1 (R_N). If there are degraded parts in the inventory and a replacement part needs to be pulled, then a good part is pulled from inventory if $R_N > \frac{G_{ik}}{NP_{ik}}$

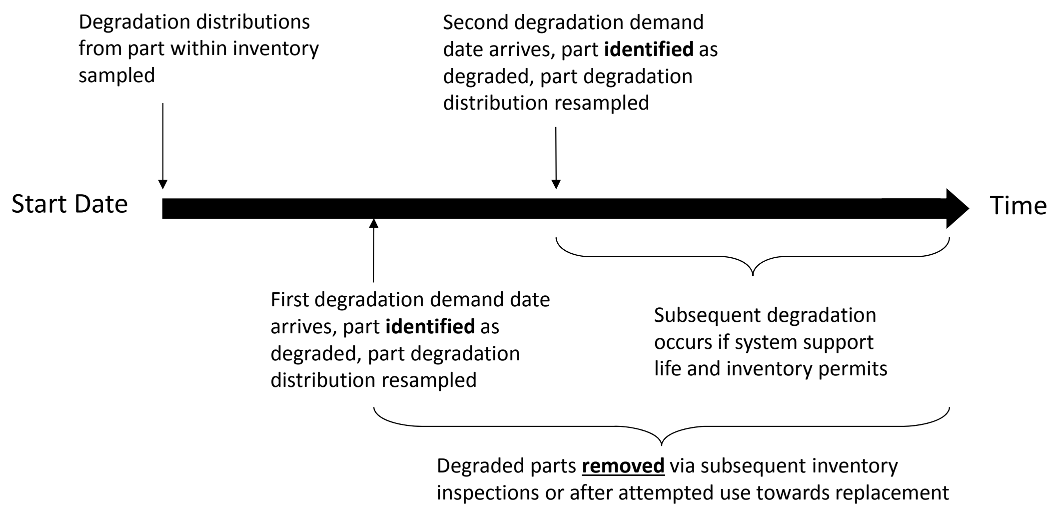


Fig. 3 Part degradation in inventory (shelf-life) process flow for a single part.

or a degraded part is pulled from inventory, $R_N < \dfrac{G_{ik}}{NP_{ik}}$.

The quantities (G_{ik}, NP_{ik}) are updated after the part is drawn (whether good or degraded) and the simulation continues. The degradation process flow is shown in Fig. 3.

The process flow in Fig. 3 demonstrates the degradation for a single part within a single inventory. In the event that multiple parts degrade, additional process flows are added in parallel and are associated with each part involved (each process flow is associated with a single part located in a specific inventory). The EOM model allows for part degradation probability distributions to be defined for each part appearing in specific part inventories within the system.

Additionally, the EOM model models periodic inventory inspections for all obsolete parts located in a given inventory. In the event of an inventory inspection, a pre-specified quantity of each part is randomly drawn from inventory; any degraded parts drawn during inspection are immediately discarded and the fully functional parts may or may not be returned to the inventory after inspection occurs. Inspections continue to occur periodically until either all the spares have been depleted from the inventory or EOM occurs.

3 Test Case and Results

To demonstrate the model, a test case was developed using an actual legacy electronic system. The system used in this study is comprised of unique cards, each card containing unique parts, and observed part failure histories. The objective of the test case is twofold: 1) to demonstrate the capability of the model, and 2) to observe the legacy system sustainment ramifications through different test scenarios (part harvesting and immediate first failures of non-failed parts).

The case considered contains 117,000 system instances each consisting of 70 unique cards totaling 4.5 million obsolete parts. Each card has a unique number of fielded units and number of available spare cards to draw from. The system was introduced in 1993, the EOM analysis begins on January 1, 2011, and system support is continuing for the foreseeable future.

The legacy system was examined using five different management assumptions representing 'worst-case' and 'best-case' scenarios (with regards to parts with no failure history) for the system and incorporating the use of part harvesting towards system sustainment. The 'best-case' scenario assumes that parts with no previous failure history within the system never fail and are therefore not considered during EOM analysis. This assumption may be valid depending on the nature of the system and when the system was introduced. The 'worst-case' scenario assumes that parts with no previous failure history experience their first failure at the beginning of the analysis and their failure distributions are synthesized based on this assumption. Each test case was tracked for 500 system life histories to construct probability distributions of EOM dates. The analysis ignored parts that were not obsolete at the analysis start date and inventories of spare cards were included in all test cases and used before inventories of accumulated harvested parts were considered. The following five test cases are considered:

Run to the first EOM in the system:

1. Best case – no harvesting
2. Worst case – no harvesting
3. Worst case – with harvesting

Run until every card has reached its first EOM (or 2050 max):

4. Worst case – no harvesting
5. Worst case – with harvesting

The first three test cases were analyzed until the first EOM date for the entire system was reached (i.e., the first instance that a part demand could not be fulfilled from available inventories). Test cases 4 and 5 sustain the system until one of two conditions was met: 1) until every card type within the system has observed its first EOM date, or 2) until the year 2050 has been reached. In test cases 4 and 5, the first condition was never met so the simulation ran to 2050 and recorded all the EOM events that occurred up to 2050. Test cases 4 and 5 were also ordered to organize EOM events and calculate associated means and probabilities on a card-level rather than system-level. This means that probability distributions of EOM dates were analyzed for individual cards rather than as a representation of the entire system (by order of occurrence).

3.1 Parts Containing Significant Failure Histories With Right Censored Failure Data. A large number of failures have been observed to date for several specific parts (e.g., 3798-05 and 5004-02) in the example system. For these parts, the time to failure distributions can be determined using Maximum Likelihood Estimation (MLE) to find the best fit to 2-parameter Weibull distributions while accounting for the surviving parts using life data analysis software (Weibull++®).[7] The resulting failure distributions for these parts are shown in Figs. 4 and 5. The two lines on each graph represent the fitted failure distributions with and without right-censoring.

[7] The failure data is right censored because not all the fielded parts have failed to date. Right censoring occurs in reliability testing when some of the units in the population survive the full test time period without failing.

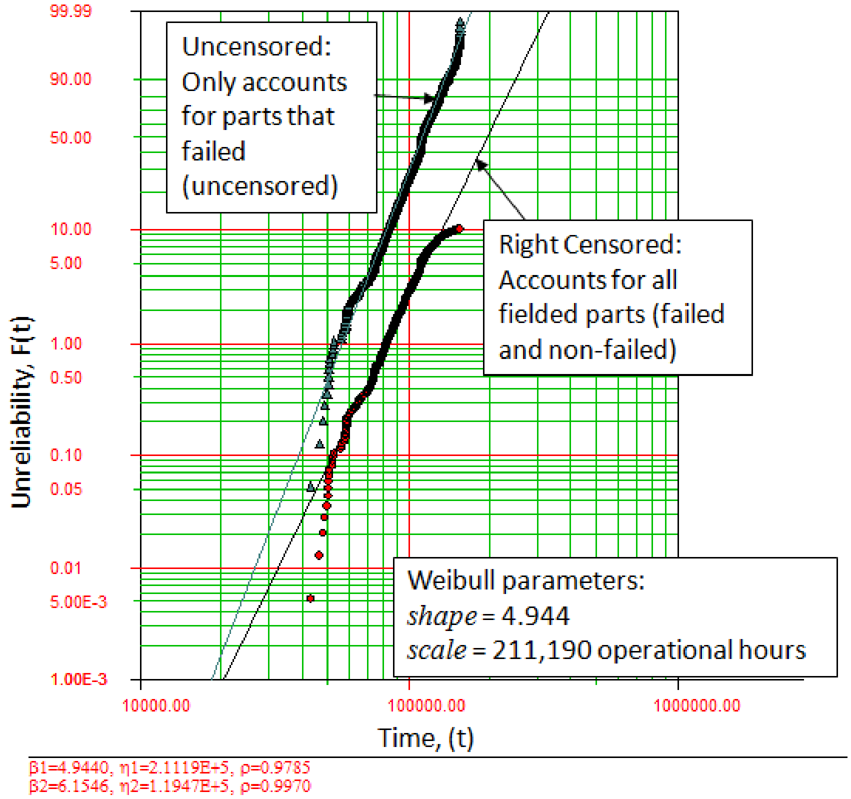


Fig. 4 Part number 3798-05 failure distribution. Both data sets are equal, one shows 10% unreliable, the other 100% unreliable (censored versus uncensored).

Other obsolete parts included in the system had too few recorded failures to make MLE fitting practical and their failure distributions were therefore treated as uniform distributions created from historical failure data as described in equations (1) and (2). Note, treatment of these parts using uniform distributions is only a default; if information is known about the time-to-failure distributions for these parts from other sources, e.g., qualification testing, it could have been used.

3.2 No Failure of Non-failed Parts and No Harvesting (Test Case 1) Results. The results for the first test case are shown in Fig. 6. The mean time to the first EOM date for the system was 17.49 years (2028.49). The left side of the figure shows a probability distribution of the first EOM dates for the system, which includes possible EOMs from all cards in the system. On the basis of running 500 system life histories of the system, that the following conclusions can be drawn:

- 50% probability that at least one instance of the system will be unsupportable by 2028.52

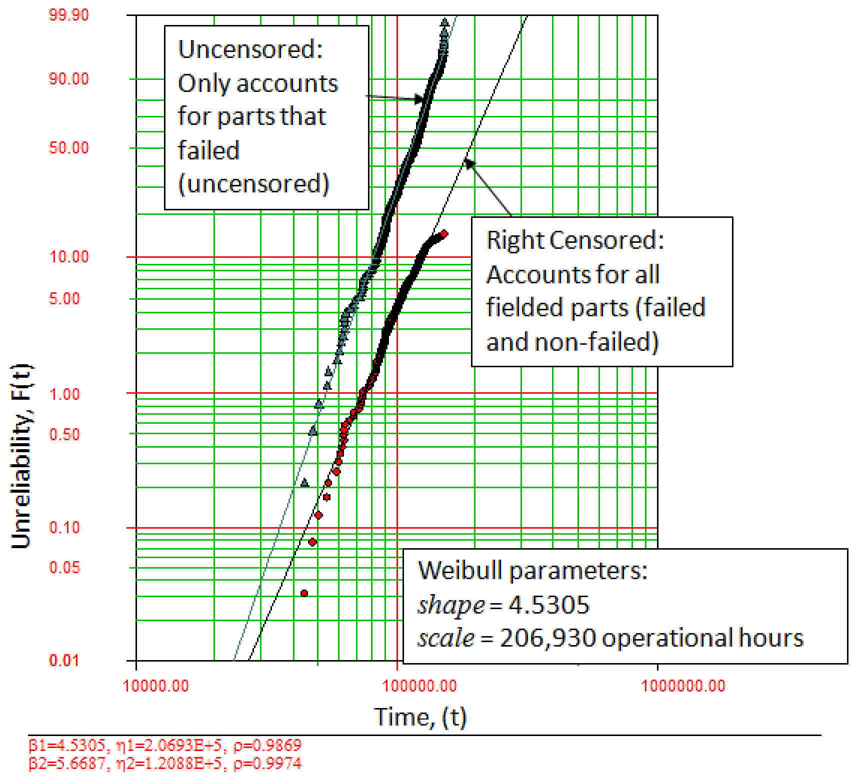
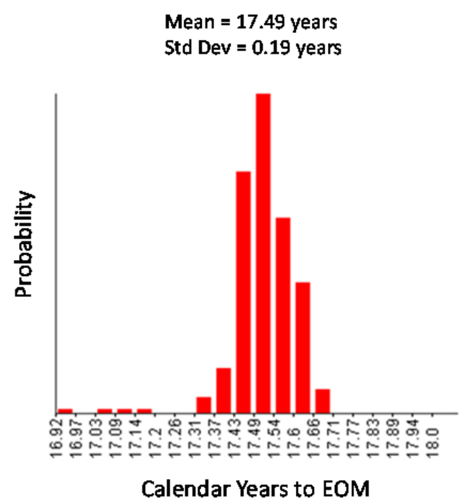


Fig. 5 Part number 5004-02 failure distribution. Both data sets are equal, one shows 10% unreliable, the other 100% unreliable (censored versus uncensored).

- ~100% (95.4%) probability of all instances of the system being supportable through the end of 2027
- 100% probability that at least one instance of the system will be unsupportable by 2029

In this particular case, the EOM date for the system is very well defined, i.e., in nearly all (95.4%) of the life histories analyzed, the system will survive without an EOM through the end of 2027, but there is a 100% probability that at least one instance of the system will be unsupportable by 2029. The right side of Fig. 6 shows the cards that are the most probable causes of EOM and EOR events. The system-level EOM distribution (left side of Fig. 6) corresponds to all observable EOMs (including parts 6763-24, 5004-02, 4000-44, and 6006-51). The part that is most likely to result in the first-ordered EOM is part 6763-24 from Card 63 (81.6%) with a mean EOM time of 17.5 calendar years. This probability means that in 408 out of 500 life histories, the 6763-24 parts from Card 63 caused the first EOM in the system.

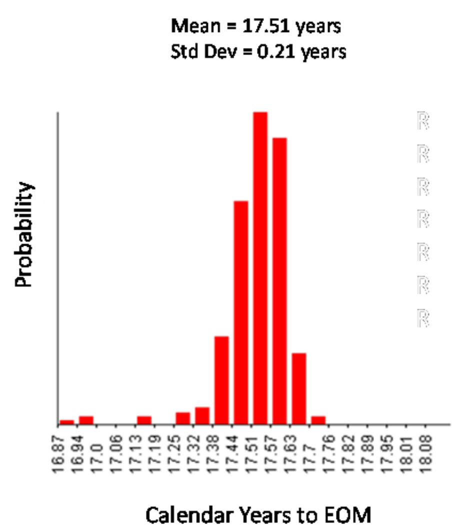


Part ID	Card ID	Mean EOM Time	Probability
6763-24	Card 63	2028.528	81.6%
5004-02	Card 61	2028.496	11.6%
4000-44	Card 30	2027.62	3.6%
6006-51	Card 41	2028.12	3.2%

Part ID	Repair Action	Mean EOR Time	Probability
4000-44	RPB Inventory	2025.311	66.4%
6763-24	RPB Inventory	2020.11	16.0%
6006-51	RPB Inventory	2025.719	8.4%
1788-63	RPB Inventory	2024.618	4.8%
4000-44	Card Stock	2027.799	2.8%
6006-51	Card Stock	2027.744	1.2%

Fig. 6 System-level EOM distribution (left), EOM (top right) and EOR (bottom right) results for Case 1.

3.3 Immediate First Failure of Non-Failed Parts and No Harvesting (Test Case 2) Results. The results for the second test case are shown in Fig. 7. The mean time to the first EOM date for the system was approximately 17.51 years (2028.5). The left side of the figure shows a probability distribution of the first EOM dates for the system. The right side of Fig. 7 shows the tabulated results of the most probable causes of EOM and EOR events in the system.



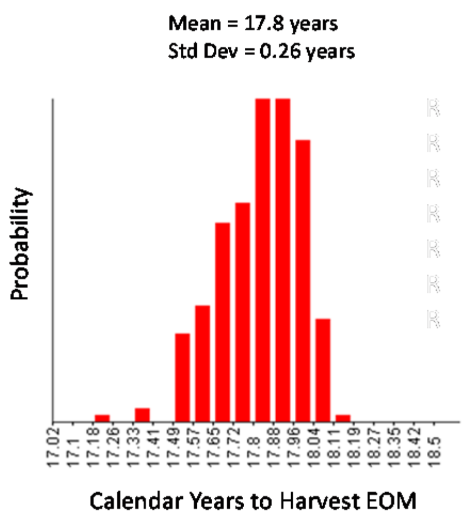
Part ID	Card ID	Mean EOM Time	Probability
6763-24	Card 63	2028.539	82.8%
5004-02	Card 61	2028.522	12.4%
4000-44	Card 30	2027.61	2.4%
6006-51	Card 41	2028.171	2.0%
4000-44	Card 57	2028.408	0.4%

Part ID	Repair Action	Mean EOR Time	Probability
4000-44	RPB Inventory	2025.298	63.2%
6763-24	RPB Inventory	2020.06	16.0%
6006-51	RPB Inventory	2025.79	11.2%
1788-63	RPB Inventory	2025.018	4.4%
4000-44	Card Stock	2027.957	3.2%
6006-51	Card Stock	2027.117	2.0%

Fig. 7 System-level EOM distribution (left), EOM (top right) and EOR (bottom right) results for Case 2.

The first End of Maintenance date does not decrease as a result of the 'worst-case' assumption for obsolete parts with no failure histories. This signifies to the system supporter that, even in a worst case scenario, the non-failed parts are not at risk for causing significant (if any) differences in system-level EOM dates.[8]

3.4 No Failure of Non-failed Parts and Harvesting (Test Case 3) Results. The results for the third test case are shown in Fig. 8. The mean time to the first EOM date for the system was approximately 17.8 years (2028.8)—resulting in a quarter year gain in system sustainment due to the action of harvesting of parts.



Part ID	Card ID	Mean EOM Time	Probability
5004-02	Card 61	2028.795	78.8%
6763-24	Card 63	2027.013	16.8%
6006-51	Card 41	2028.115	4.4%

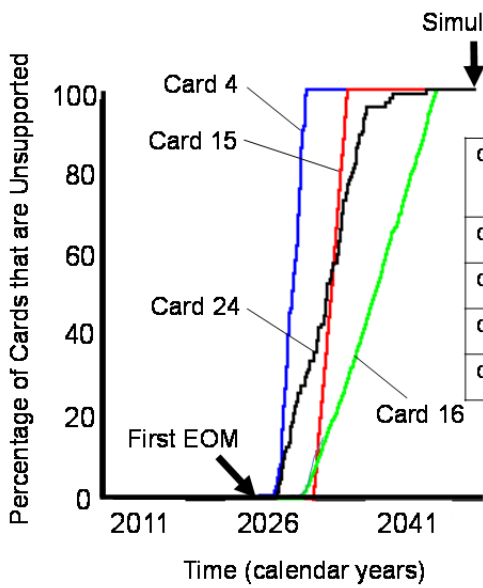
Part ID	Repair Action	Mean EOR Time	Probability
4000-44	RPB Inventory	2025.33	65.6%
6006.51	RPB Inventory	2026.246	18.4%
6763-24	RPB Inventory	2025.072	7.6%
1788-63	RPB Inventory	2025.062	4.8%
4000-44	Card Stock	2027.654	1.6%
6006-51	Card Stock	2028.115	1.2%

Fig. 8 System-level EOM distribution (left), EOM (top right) and EOR (bottom right) results for Case 3.

All the parts that cause the first EOM event when part harvesting is implemented also appear in previous cases 1 and 2. One of the part-card combinations that caused the first EOM event in the previous case is now delayed. Furthermore, when the difference in the mean is evaluated between cases 1 and casess 3, it is shown that the mean increase as a result of is statistically significant when compared to the coefficient of variation.

[8] The coefficient of variation (COV) for Case 1 is 0.01—to determine statistically significant deviations from the mean, a number for comparisons must fall outside this range. When the difference between the means of Case 1 and 2 are calculated, it is determined that, although small, there is statistical significance between the two Cases.

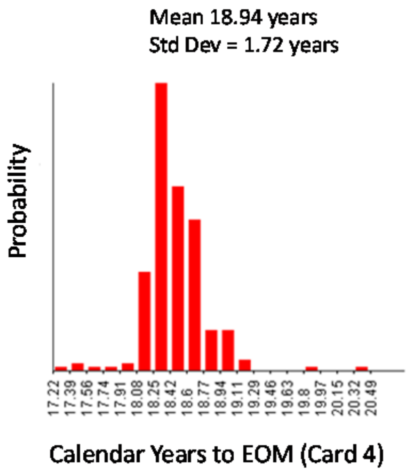
3.5 Immediate First Failure of Non-failed Parts and No Harvesting (Test Case 4) Results. In this case the analysis ran until the year 2050, tracking all observed EOM events. The EOM model can track specific cards throughout the system lifecycle detailing how the fielded cards were removed over time due to EOM events. Each of the tracked cards shown in Fig. 9 become unsupportable (characteristic when all fielded cards are removed due to the failure to meet part demands) by specific calendar dates. The card-level EOM results for the fourth test case can be seen in Fig. 10.

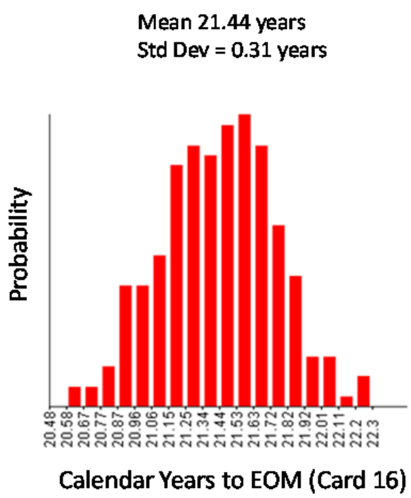


Card ID	Date when all fielded instances of the card are removed due to failure to meet part demands
Card 4	2032.3
Card 15	2036.6
Card 24	2044.9
Card 16	2045.9

Fig. 9 Card-level support tracking and loss.

The left graphs in Fig. 10 show the card-level EOM probability distributions for specific cards in the system. The table on the right side of Fig. 10 shows a list of the cards within the system that observed at least one EOM event up until the calendar date (2050) when the simulation was terminated. It was shown that 41 out of the 70 cards in the system (22 cards are shown in Fig. 10) exhibited first EOM dates prior to 2050 and probability distributions for each card that experienced EOM can be provided.





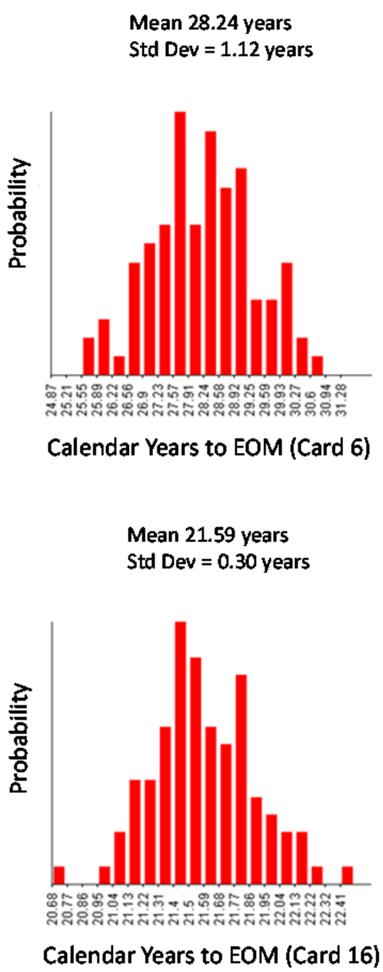
Part ID	Card ID	Mean EOM Time	Probability
3002-89	Card 1	2038.93	100.0%
2000-04	Card 2	2033.677	100.0%
8971-33	Card 3	2040.943	100.0%
3834-35	Card 4	2029.941	100.0%
1313-88	Card 5	2033.624	100.0%
0048-02	Card 6	2034.917	100.0%
5025-07	Card 7	2037.781	48.8%
7979-66	Card 8	2035.702	100.0%
3985-00	Card 9	2039.394	100.0%
8282-04	Card 10	2035.365	100.0%
5362-13	Card 11	2029.686	40.8%
9398-55	Card 12	2048.817	86.4%
1723-00	Card 13	2034.625	100.0%
6347-27	Card 14	2048.333	78.4%
6763-24	Card 15	2033.515	100.0%
5004-02	Card 16	2032.438	100.0%
5322-78	Card 17	2033.583	100.0%
5890-74	Card 18	2030.658	100.0%
6006-51	Card 22	2034.376	100.0%
3118-66	Card 23	2035.823	100.0%
4000-44	Card 24	2028.473	100.0%
4000-60	Card 25	2037.753	100.0%

Fig. 10 Card-level EOM distributions (left) and EOR/EOM results for Case 4 (right).

3.6 No Failure of Non-failed Parts and Harvesting (Test Case 5) Results. The results for this test case are shown in Fig. 11, the only difference between cases 4 and 5 is the inclusion of part harvesting. The case 5 results differ from case 4 in that 22 of the 70 cards in the system exhibited first EOM dates prior to 2050 without harvesting and 19 of the 70 cards exhibit a first EOM prior to 2050 when harvesting is used.

Part harvesting allows for some card-level EOM dates to be delayed for significant periods of time in this example case. This result depends on many factors including parts' failure distributions and whether critical parts that cause card-level EOM events appear on multiple cards within the system. However, the action of harvesting parts may not significantly delay card-level EOM dates when faced with high failure rate parts (due to an excessive number of demands at a given time and lack of supply).

By comparing Fig. 10 and Fig. 11, one can see the added benefit (or lack thereof) when harvesting is implemented. Card 16 was selected in Figs. 10 and 11 to show that harvesting sometimes may not prove fruitful; in other cases, such as Card 4, Case 5increases the mean EOM date by 12 years compared to Case 4.



Part ID	Card ID	Mean EOM Time	Probability
3687-32	Card 1	2040.258	100.0%
2000-04	Card 2	2034.572	100.0%
5362-13	Card 3	2042.956	100.0%
6006-51	Card 4	2041.642	95.0%
1628-68	Card 5	2035.631	100.0%
1438-85	Card 6	2039.244	100.0%
4022-14	Card 7	2042.208	95.0%
5503-89	Card 8	2038.164	100.0%
3985-00	Card 9	2042.335	95.0%
9074-13	Card 10	2039.843	100.0%
6262-95	Card 11	2043.962	100.0%
9398-55	Card 12	2049.859	43.7%
1723-00	Card 13	2031.952	18.0%
6347-27	Card 14	2048.018	100.0%
3798-05	Card 15	2034.301	100.0%
5004-02	Card 16	2032.559	100.0%
1818-11	Card 17	2034.437	100.0%
2008-84	Card 18	2038.71	83.0%
6006-51	Card 22	2036.528	100.0%
8897-53	Card 23	2034.627	40.3%
4000-44	Card 24	2028.78	98.7%
4000-60	Card 25	2047.68	52.6%

Fig. 11 Card-level EOM distributions (left) and EOM results for Case 5 (right).

3.7 Selective Card Refresh. It may be possible to extend the EOM dates of systems by design refreshing selected cards. A design refresh refers to the replacement of one or more obsolete parts with non-obsolete parts, in order to keep the card sustainable. The refresh of specific cards will extend the EOM date associated with the card (and may thereby possibly extend the system's EOM date) and/or refreshing a card may allow the harvesting of parts (from the refreshed card) for supporting other cards that use the same parts.

In this section, a sensitivity analysis is conducted for the previous test cases 1-3 to determine the additional system support life gained from selectively implementing a design refresh for individual cards in the system. The completion dates for the individual card refreshes for each case were determined from their respective EOM date probability distributions (see Figs. 6-8). The completion dates for the selective card refreshes are implemented to occur prior to the earliest EOM date observed for the respective test case (i.e., the earliest EOM date observed for case 1 was 2027.9, and the selective design refreshes for case 1 are assumed to be complete in 2027). This assumption implements design refreshes at the latest possible time—resulting in less expensive (cost of money) design refreshes, and the possibility of refreshing additional obsolete parts that may become obsolete over the support life of the system. The completion dates for the individual card refreshes for cases 1-3 are shown in Table 1.

Table 1 Completion dates for implemented individual card refreshes

Case #	Individual Refreshed Cards	Card Refresh Completion Date
1	30, 41, 61, 63	2027
2	30, 41, 57, 61, 63	2028
3	All	2028.5

The individual cards chosen for design refreshes for each test case were determined first by the identified cards that potentially caused the first EOM date for the system (for test cases 1 and 2). Additional cards were also chosen and tested individually when they were identified to cause the first EOM date for the system as a result of implemented individual card refreshes. Selective design refreshes for case 3 included testing all cards within the system, as harvested parts can potentially be used to support other cards in the system. The results for each test case and their selected cards are seen in Figs. 12 through 14. The results shown include individual refreshes for which there was a statistical difference among the means between the individual card refresh first EOM dates and the first EOM date from each test case from Figs. 6-8. In order to determine whether or not there was any statistical difference among the means, a two sided-hypothesis test was performed using the mean EOM date when a selected design refresh was introduced and the mean EOM date for the associated test case (best, worst, worst case including harvesting) where design refresh was not introduced.

The two-sided hypothesis testing is given by,

$$H_0\text{: } \mu_{DR} = \mu_{TC} \tag{9}$$

$$H_1\text{: } \mu_{DR} \neq \mu_{TC} \tag{10}$$

where,

H_0 = null hypothesis

H_1 = alternate hypothesis

μ_{DR} = population mean for a selected design refresh

μ_{TC} = population mean for the associated test case (no design refresh).

The EOM date probability distributions were assumed to approximate normal distributions based on the Central Limit Theorem, which states as the sample size becomes larger, the population frequency distribution approximates a normal distribution. A confidence level of 95% (α = 0.05) was used for each investigated case, and the selected refreshes that were shown to be statistically different (able to reject H_0) are shown in Figs. 12 through 14.

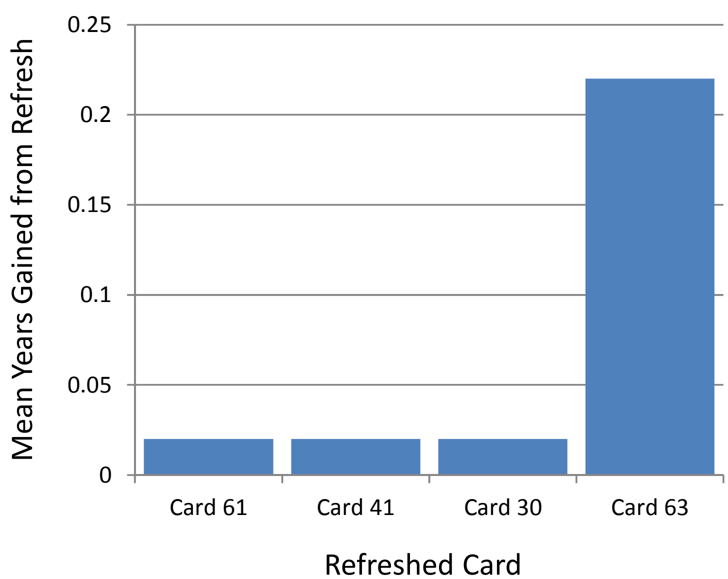


Fig. 12 System support life gained from individual card refresh, case 1.

The results from case 1 show that there were four individual cards that had a statistically different mean system support life via design refresh compared to case 1. The best candidate for design refresh was Card 63 granting a system extended life of 0.22 years.

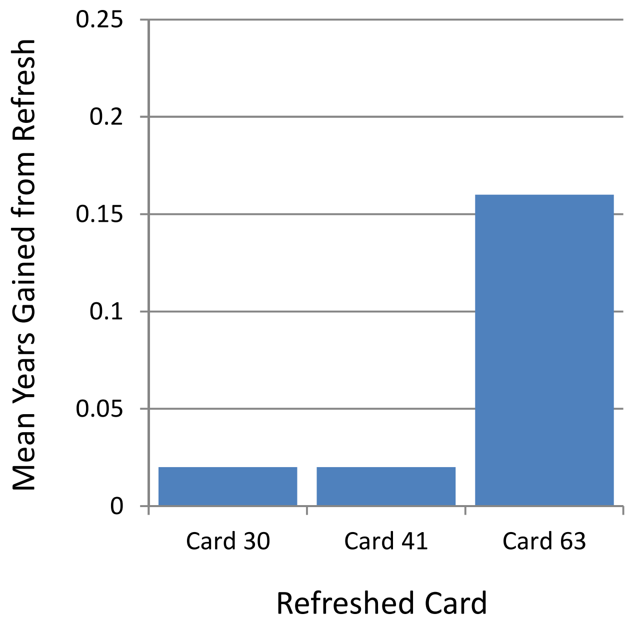


Fig. 13 System support life gained from individual card refresh, case 2.

The results from case 2 show that there were three individual cards that had a statistically different mean system support life via design refresh compared to case 2. The best candidate for design refresh was Card 63 granting a system extended life of 0.16 years.

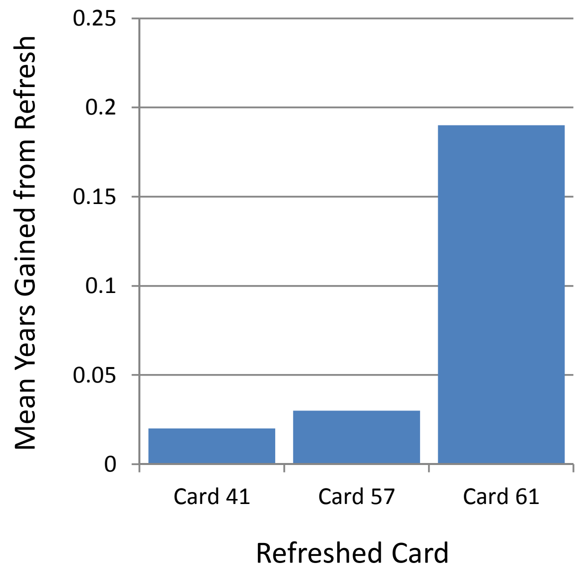


Fig. 14 System support life gained from individual card refresh, case 3.

The results from case 3 show that there were three individual cards that had a statistically different mean system support life via design refresh compared to case 3. The best candidate for design refresh was Card 61 granting a system extended life of 0.19 years.

3.8 Design Refresh Planning. Design refreshes may also be performed on selected cards to ensure system sustainment to a specific date. In this context, the EOM model assumes that a design refresh is completed for an individual card on the date that it is *necessary* (i.e., the first EOM date for that particular card) and records the completed date of the refresh and the identity of refreshed card. The following analyses assume a maximum of one design refresh per type of card—all the obsolete parts on the refreshed card are removed and excluded from the analysis for all times after the refresh is completed. The EOM model was used to determine design refresh plans to ensure system sustainment until the year 2050 for the first three test cases. The refreshes during design refresh planning are assumed to be completed "just-in-time" on the earliest EOM date associated with each type of card.[9] This assumption implements design refreshes at the latest possible time—resulting in less expensive (cost of money) design refreshes, and allows for design refresh planning (design refresh as needed) rather than selectively entering design refreshes at specified dates.

[9] This is the same as the "just in time refresh date" assumption made in general refresh planning, [8].

The results from case 1 show that there were 10 individual cards generated in the design refresh plan to ensure system sustainment to the year 2050. The probabilities indicate the likelihood that a given card would undergo a design refresh based on the number of system life histories that were simulated. The design refresh plan and a probability distribution of completion dates for individual refreshed cards are shown in Fig. 15.

Card ID	Mean Refresh Completion Date	Probability
Card 41	2033.572	100.0%
Card 63	2033.494	100.0%
Card 61	2032.452	100.0%
Card 39	2047.228	63.0%
Card 31	2044.837	75.4%
Card 32	2044.634	71.0%
Card 57	2036.629	100.0%
Card 29	2042.602	90.4%
Card 30	2034.271	99.8%
Card 40	2043.655	97.8%

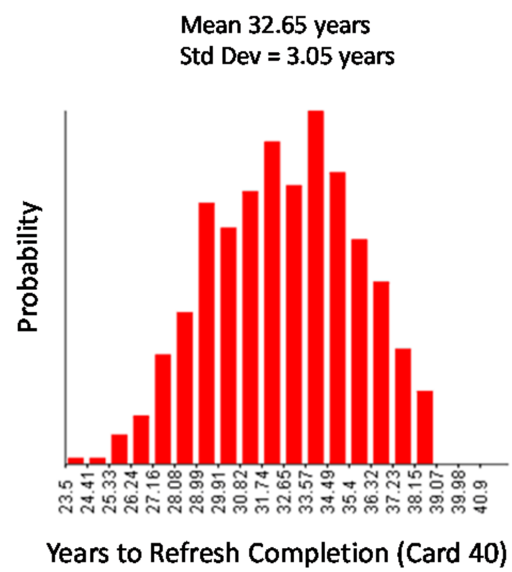


Fig. 15 Design refresh plan (left) and completed refresh date distribution (right), case 1.

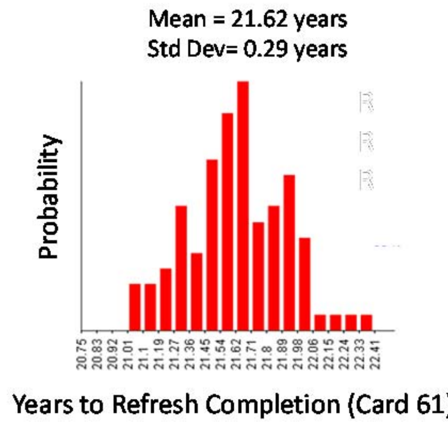
The results from case 2 show that there were 39 individual cards generated in the design refresh plan to ensure system sustainment to the year 2050. The design refresh plan is seen in Fig. 16. The introduction of the immediate failure of obsolete parts with no failure histories led to the increase in the failure of the additional cards prior to 2050, requiring them to be design refreshed.

Card ID	Mean Refresh Completion Date	Probability	Card ID	Mean Refresh Completion Date	Probability
Card 4	2038.912	100.0%	Card 64	2048.25	81.4%
Card 59	2033.677	100.0%	Card 63	2033.489	100.0%
Card 14	2040.466	100.0%	Card 61	2032.457	100.0%
Card 18	2033.599	100.0%	Card 39	2030.75	100.0%
Card 50	2040.971	100.0%	Card 31	2030.381	100.0%
Card 12	2032.647	100.0%	Card 32	2030.402	100.0%
Card 41	2029.366	100.0%	Card 54	2033.627	100.0%
Card 22	2033.541	100.0%	Card 15	2037.023	100.0%
Card 21	2037.006	100.0%	Card 57	2030.038	100.0%
Card 42	2029.573	100.0%	Card 29	2030.146	100.0%
Card 48	2043.173	100.0%	Card 30	2029.469	100.0%
Card 3	2035.692	100.0%	Card 40	2030.255	100.0%
Card 25	2032.863	100.0%	Card 20	2048.438	72.6%
Card 24	2035.141	100.0%	Card 1	2042.682	100.0%
Card 49	2035.341	100.0%	Card 19	2033.759	100.0%
Card 13	2029.627	100.0%	Card 17	2034.596	100.0%
Card 55	2037.562	100.0%	Card 16	2034.445	100.0%
Card 62	2034.602	100.0%	Card 21	2039.545	100.0%

Fig. 16 Design refresh plan, case 2.

The results from case 3 show that there were 22 individual cards generated in the design refresh plan to ensure system sustainment to the year 2050. The design refresh plan and a probability distribution of completion dates for individual refreshed cards are shown in Fig. 17. The implementation of part harvesting reduced the required design refresh plan for case 2 by 17 cards.

Card ID	Mean Refresh Completion Date
Card 4	2043.182
Card 59	2034.657
Card 50	2041.634
Card 12	2038.21
Card 41	2033.915
Card 23	2040.03
Card 56	2041.305
Card 6	2035.57
Card 48	2041.04
Card 3	2040.69
Card 25	2045.361
Card 49	2038.741
Card 13	2037.087
Card 55	2038.228
Card 62	2034.939
Card 63	2034.672
Card 61	2032.624
Card 54	2034.419
Card 57	2036.236
Card 7	2034.584
Card 1	2044.713
Card 16	2035.107



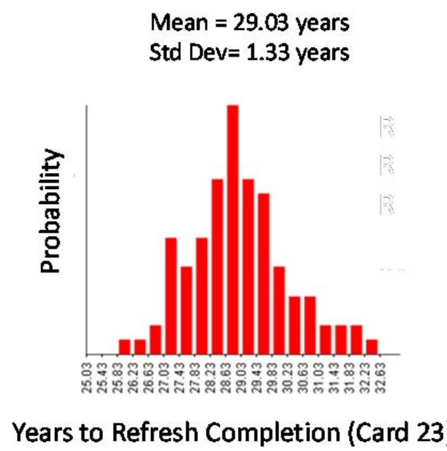


Fig. 17 Design refresh plan (left) and completed refresh date distributions (right), case 3.

3.9 Summary of Case Study Results. The results from the case study scenarios used to demonstrate the methodology and capabilities of the EOM model are summarized in Table 2.

Table 2 Case Study Summary

Case(s)		Mean First EOM	First EOM Events by 2050	EOM Extensions Possible Through Individual Card Refresh	Design Refresh Plans Necessary to Ensure System Sustainment Through 2050
1	Best-case, no harvesting	17.5 calendar years		0.22 calendar years (Card 63)	10 individual card refreshes
2 and 4	Worst-case, no harvesting	17.5 calendar years	41 individual cards	0.16 calendar years (Card 63)	39 individual card refreshes
3 and 5	Worst-case, with part harvesting	17.8 calendar years	22 individual cards	0.19 calendar years (Card 61)	22 individual card refreshes

The test case scenarios included results for an actual legacy electronic system using the harvesting of parts, immediate first failure assumption for no-failure obsolete parts, and system sustainment to a specified end of support date to track subsequent EOM events. The model predicted that the electronic system used in the case study would be able to last between 17.5 and 17.8 years as shown in the third column of Table 2. The immediate first failure assumption for no-failure obsolete parts did not reduce the system support life capabilities meaning that non-failed parts would not be an issue for causing EOM. The implementation of part harvesting extended the system support life by approximately 0.3 years. Therefore, the activity of part harvesting for the case study system resulted in an extension of the overall support life of the system by approximately 2%.

The EOM model was then used to track subsequent EOM events in order to sustain the electronic system to the year of 2050. The model predicted that the system used for the case study would incur first EOM events as shown in the fourth column of Table 2. In this test case, the implementation of harvesting led to an avoidance of 19 additional cards incurring EOM events by the year 2050—showing that there is part similarities among the cards within the electronic system, and that part harvesting is a significant mitigation strategy for delaying additional EOM events for systems whose cards have part similarities.

The EOM model was also used to observe the effects of individual selected card refreshes on system sustainment. The model predicted that the system used for the case study could extend its average first EOM date as shown in the fifth column of Table 2. The extension of the system support life is most extended through the individual card refresh of the Card 63 when part harvesting is not considered. This is due to the fact that Card 63 has a variety of components that have high demand rates (low reliabilities) caused by unique parts that fail frequently and do not appear on other assemblies. However, the system support life is most extended through the individual card refresh of Card 61 when part harvesting is used. Therefore, this shows that Card 61 has the most (advantageous) part dependencies among other assemblies that when Card 61 is design refreshed its components can be used as spares for other assemblies with similar components.

The EOM model was also used to produce design refresh plans, in order to ensure system sustainment to a specified end of support date. The model predicted that design refresh plans to ensure system sustainment until the year 2050 for the case study system include between 10 and 39 individual card refreshes as shown in the sixth column of Table 2. The implementation of harvesting led to an avoidance of 19 additional cards incurring EOM events by the year 2050—showing that there is part similarities among the cards within the electronic system, and that part harvesting is a viable tactic for delaying additional EOM events for systems whose cards have part similarities. Design refresh planning (worst-case) delayed 2 EOM events (compared to worst-case of tracking subsequent EOMs to 2050), due to eliminating the additional part demands via design refreshing.

4 Discussion and Conclusions

The business of supporting legacy electronic systems is challenging due to mismatches between the system support life and the procurement lives of the systems' constituent components. In this paper we describe an End of Maintenance (EOM) model, which is a stochastic discrete-event simulation that follows the life history of a population of parts and cards and determines how long the system can be sustained based upon existing inventories of spare parts and cards, and optionally harvesting of parts off of existing cards to increase system support life.

The model described in this paper assumes "standard" parts. DMSMS-type obsolescence problems in electronic systems also occur for non-standard (i.e., altered or programmable) parts. Non-standard parts create several additional issues in the analysis of End of Maintenance. First, there may not be a one-to-one correspondence between non-standard parts and the spares inventories from which they are drawn. Multiple unique non-standard parts may be drawn from a single inventory item, e.g., unprogrammed FPGAs (Floating Point Gate Arrays) may be used to create more than one unique part, or conversely a single non-standard part may need to draw multiple items from multiple inventories. A second issue is that the definition of DMSMS-type obsolescence for non-standard parts is not be clear.

The model could be extended to include non-obsolete parts that will become obsolete during the analysis period. Obsolescence forecasting for electronic piece-parts is well developed and commercially available (e.g., [22-24]); forecasts could be used to determine when parts that are not obsolete at the start of the analysis will become obsolete. However, such analysis requires a model (or policy) for how obsolescence is mitigated when these parts become obsolete, i.e., if lifetime buys are made, how buy quantities determined and what dates they are purchased for.

The EOM model described in this paper provides the legacy system supporter with insight about how long the system can be supported based on existing inventories of spare parts and cards including the critical identification of parts/cards (and their associated likelihoods) that cause system support loss. The resulting forecasted system support life and its associated levels of uncertainty allow system supporters to make appropriate decisions concerning system sustainment and possible system replacement. End of Maintenance analysis may be necessary in order to make a business case for replacing a legacy system effectively has no end of support date (or for which end of support dates are regularly extended).

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Nomenclature

a	=	lower bound of a uniform distribution
AFD_{im}	=	adjusted forecasted demand of the *i*th part from the *m*th card
b	=	upper bound of a uniform distribution
D_A	=	date (in years) of the start of analysis
D_{FF}	=	date (in years) of the first failure
D_S	=	date (in years) the part was fielded
DD_{ik}	=	*i*th part forecasted degradation demand date from the *k*th inventory
FD_i	=	*i*th part forecasted demand date
FD_{im}	=	*i*th part forecasted demand date from the *m*th card
FDD_{ik}	=	forecasted degradation date for the *i*th part from the *k*th inventory
G_{ik}	=	number of *i*th parts considered degraded from the *k*th inventory
H_i	=	preserved life fraction of *i*th part incurred from the physical action of harvesting (1 = no damage to the part from the harvesting process, 0 = all remaining life removed by the harvesting process)
k	=	number of life histories simulated
L_{ij}	=	life removed from the *i*th harvested part from the *j*th card
$\overline{M}_i$	=	*i*th-ordered mean EOM time
M_{ij}	=	*i*th-ordered EOM time in the *j*th life history

N_f	=	number of failures to date
N_{ij}	=	number of occurrences as an *i*th-ordered EOM in the *j*th life history - either 1 (occurs) or 0 (does not occur)
N_T	=	total number of fielded parts within entire system
NP_{ik}	=	number of *i*th parts remaining from the *k*th inventory
O_p	=	operational hours per year
P_i	=	*i*th-ordered EOM probability
t	=	current simulation time (starting at $t = 0$)
t_H	=	simulation time when the harvesting action occurs
t_i	=	simulation time when *i*th part was fielded (t_i=0 for initial fielded parts)

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