Safety and Reliability for Managing Risk – Guedes Soares & Zio (eds)
© 2006 Taylor & Francis Group, London, ISBN 0-415-41620-5

Forecasting the cost of unreliability for products with two-dimensional warranties

A. Kleyner
Delphi Corporation, Electronics & Safety Division, Kokomo, Indiana, USA

P. Sandborn
CALCE Electronic Products and Systems Center, Department of Mechanical Engineering, University of Maryland, College Park, Maryland, USA

ABSTRACT: This paper addresses the effect of two- dimensional warranty policies on the procedure for forecasting the cost of unreliability. Automotive warranties are characterized by age or time in service and vehicle usage mileage. This paper presents a model where the usage time is a primary variable and the mileage accumulation is estimated from field return data. At each time interval the probability of exceeding the upper mileage limit on the warranty is calculated based on the mileage distribution and an analytical cumulative distribution function (CDF) of exceeding the upper mileage limit at any point of time is constructed. This modeling procedure accounts for an observed reduction in the number of warranty claims in the second half of the warranty period thus making it a more realistic evaluation of products with two-dimensional warranty policies. A case study using an automotive parts example in order to illustrate the methodology is provided.

1 INTRODUCTION

Automotive warranties amount to a whooping $12 billion per year for North American manufacturers alone (Warranty Week Newsletter 2004a). According to Warranty Week Newsletter (2004b), this amount constitutes more than half of all the warranties for all US manufacturers worldwide. Therefore, finding the best possible ways of predicting future warranty claims and more accurately accounting for the existing warranties would have a great engineering and financial impact on the whole process of planning and analyzing warranties in the automotive industry.

Warranty forecasting is now an intrinsic part of an economic model used for new business quoting and contractual agreements between automotive OEMs and their suppliers. Inaccurate estimation of future warranty returns may result in either non-competitive bidding or unprofitable business.

1.1 *Automotive Warranty Overview*

Despite the wide variety of failure modes often exhibited in warranty claims, their overall failure rates tend to follow the first two sections of the bathtub curve, as illustrated in Figure 1. Bathtub curve is an idealized plot, which consists of three regions: infant mortality, useful life, and wear-out (O'Connor 2003). Failure rates in Figure 1 are presented in the form of incidents per thousand vehicles (IPTV). IPTV is analogous to repairs per 1000 units (R1000) also used in the industry. Note: the actual IPTV values have been modified to protect the proprietary nature of the data.

In addition to the bathtub trend, most models in Figure 1 also show the downward trend in IPTV after approximately 400 days in service, which reflects a reduction in number of reported warranty claims.

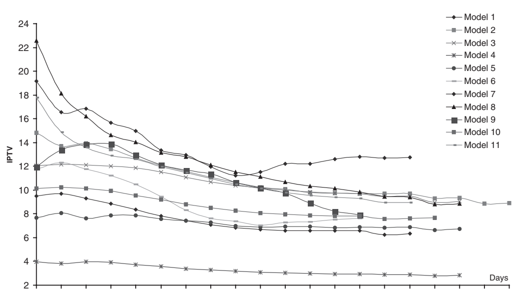

Figure 1. Failure rates expressed in IPTV for selected passenger compartment mounted electronic products recorded by Delphi Electronics & Safety. Results are plotted for 11 different vehicle models.

2 TWO-DIMENSIONAL ASPECTS OF WARRANTY

A comprehensive taxonomy for the different warranty policies is given in (Blischke & Murthy 1994). Warrantees can be broadly grouped into one- and two-dimensional policies. A one-dimensional warranty policy is characterized by a one-dimensional time interval often referred as the warranty period.

Automotive warranties in turn are often expressed in terms of both time and mileage, and fall under the category of two-dimensional warranties. Examples include a typical USA automotive warranty of 36 months or 36,000 miles (whichever comes first), or 2-yrs/50,000 km, which is typical for some European vehicles.

The literature on warranty is vast; however the bulk of it deals with one-dimensional policies. In contrast, the study of two-dimensional warranty policies received less attention. Among the published works that are relevant to automotive type two-dimensional warrantees are (Iskandar & Murthy 2003), which deals with the issue of best repair-replace strategy. (Chukova & Robinson 2004) address the issue of truncation and incomplete observation of vehicle mileages as a usage variable. (Krivtsov & Frankstein 2004, 2006) address the importance of understanding failure modes in selecting the primary usage measure between time and mileage. (Iskandar et al. 2005) analyze warranty servicing strategies considering the two-dimensional nature of usage variables. (Rai & Singh 2005) analyze the effect of change in two-dimensional warranty limits on warranty cost and possible biases of their estimates. Several other papers on two-dimensional warranties are listed in the literature reviewed by (Murthy & Djamaludin 2002).

However the issue of warranty prediction modeling using two-dimensional variables has not been sufficiently covered and the existing work lacks practical approaches that could be efficiently applied by practicing engineers rather than experienced statisticians. There is also a clear need for two-dimensional warranty analysis methods that are compatible with the existing warranty claims reporting structure and the automotive data formats – existing models are not. This paper is intended to address this gap and present a practical engineering method of accounting for the effect of mileage accumulation as a secondary usage variable in the automotive warranty forecasting process.

Mathematically, two-dimensional warranties can be specified in terms of $\{T_0, M_0\}$ with T_0 being a specified maximum time-period and M_0 a specified maximum mileage. Different shapes for the region characterize different policies. For an automotive warranty claim to occur, the two conditions must be met: $t \leq T_0$ and $m \leq M_0$, which corresponds to a rectangular region, Figure 2. Usage path 1 in Figure 2 shows the case where maximum warranty mileage M_0 is reached first and path 2 where maximum service life T_0 is reached before M_0.

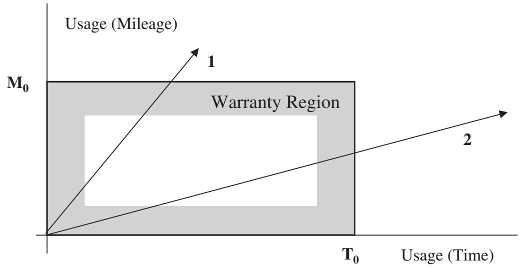

Figure 2. Warranty region for two-dimensional automotive warranty.

Even though most two-dimensional (2D) warranty policies have rectangular regions, other variations are possible, such as a triangular shape, where the boundary of that region will be defined as an arithmetic combination of time and mileage or other usage parameters analogous to cumulative damage models presented in Ebeling (1997). For more information on 2D shapes see for example Blischke & Murthy (1994) or Yang & Zaghati (2002). Higher dimensional warranties are also possible, but they are not common.

Age is known for all sold vehicles all the time, but mileage is only observed for a vehicle with a claim and only at the time of the claim. However, for automotive parts it is more common to use time as the primary usage variable due to warranty data formats and other convenience issues related to the fact that the mileage of non-failed vehicles in the field is unknown. Therefore, certain assumptions need to be made about the mileage accumulation in order to be able to properly account for the 2D aspects of an automotive warranty.

Mathematically the product failures are often described by the failure probability density function $f(t)$, thus the unreliability or one-dimensional (time only) cumulative distribution function at time T would be:

$$F(T)_{Time-based} = \int_0^T f(t)dt \tag{1}$$

where $F(t)_{Time\text{-}based}$ = cumulative distribution function of time, sometimes referred as *failure probability function*.

Failure probability function can be linked to IPTV by equation (2) below:

$$F(t) = \frac{IPTV(t)}{1000} \tag{2}$$

Table 1. Example of Automotive dealership warranty data reporting.

Vehicle identification	Production date	Sale date	Miles at claim	Claim date	Problem description
XXXX1	28-Jun 2004	13-Nov 2004	6,475	22-Apr 2005	CD will not eject
XXXX2	31-Jul 2003	27-Aug 2003	28,997	20-Jun 2005	Noisy FM reception

Where *IPTV(t)* is the total number of incidents per thousand vehicles at time t. For example, 15 IPTV will translate into 0.015 (1.5% failures).

Warranty forecasting for the products in their early stages of development or production is a common activity in many industries, where warranties take up a significant part of product lifecycle cost. There are a multitude of warranty forecasting models in the literature partially reviewed by Murthy & Djamaludin (2002). The choice of a particular warranty model can be based on failure modes, format of the data, past warranty history, and multitude of other factors. In this paper we will use the warranty forecasting model presented in Kleyner & Sandborn (2005). This model, given in (3), is based on a piecewise statistical distribution, which reflects the stabilization of the hazard rate at the time-point t_S.

$$F(t) = 1 - e^{-\left(1+\frac{\beta(t-t_S)}{t_S}\right)\left(\frac{t_S}{\eta}\right)^{\beta}} \quad t \geq t_s \tag{3}$$

where:
t_S = Hazard rate stabilization point
β = Weibull slope of the failures observed before the time t_S
η = Weibull scale parameter of the failures observed before the time t_S.

The majority of the warranty prediction models presented in the literature, including Kleyner & Sandborn (2005), do not account for mileage accumulation and use a one-dimensional usage factor, which tends to overestimate the total number of claims at the end of the warranty period. In this paper we will present an engineering procedure that allows a model correction accounting for the effect of 2D warranty policies.

Automotive dealership warranty data typically contains the dates and mileages associated with each warranty claim shown in the warranty reporting example in Table 1. Based on this information we can obtain a large number of data points with daily mileages calculated according to (4) and consequently best-fit this data into a probability distribution function for the daily mileages $f_{Daily}(m)$.

$$\text{Daily Mileage} = \frac{\text{Miles at Claim}}{\text{Claim Date - Sales Date}} \tag{4}$$

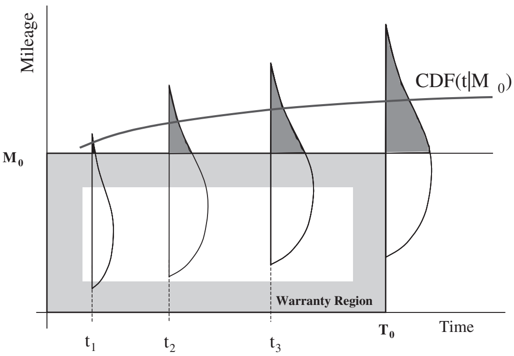

Figure 3. Vehicle mileage accumulation in two-dimensional warranty plane.

In this work, the daily mileage distribution function $f_{Daily}(m)$ was obtained from the randomly sampled dealership data of one major US vehicle manufacturer using more than 1000 data points. Each of those points was presented in a format similar to Table 1, containing the number of days to failure and the corresponding vehicle mileage.

As vehicles progress in age, the probability of exceeding the upper mileage limit M_0 is increasing as can be illustrated in Figure 3.

In Figure 3 for each time-point t_i we can calculate the cumulative probability distribution function of exceeding M_0 based on the known mileage distribution at each t_i as:

$$CDF(t_i \mid M_0) = \int_{\frac{M_0}{t_i}}^{\infty} f_{Daily}(x)dx \tag{5}$$

where t_i is expressed in days and M_0/t_i is the daily mileage required to reach M_0 at the time t_i. For each arbitrarily selected t_i the $CDF(t_i|M_0)$ can be calculated and consequently fit into an analytical function. Based on the generated sample of points t_i we can run a best fit to determine the PDF $f(t|M_0)$, a continuous function of time characterizing the probability of running out of warranty at any particular time t.

For example, for US passenger cars $f_{Daily}(m)$ was best approximated by two-parameter Weibull distribution with the shape parameter $\beta = 1.55$ and the scale

Table 2. Probabilities of exceeding 36,000 miles based on daily mileage distribution.

Time in the field, days	100	200	300	400	500	600	700	800	900	1000
Probability % of exceeding 36,000 miles	1E-11	0.0051	0.51	3.42	9.17	16.51	24.3	31.4	38.3	44.22

parameter $\eta = 41.1$, which translates into a median of 32.5 miles (52 km) and $\sigma = 24.3$ miles (39 km). The next step is to plot the probability of running out of warranty for the number of time periods t_1, t_2, t_3, etc., according to the diagram presented in Figure 3.

For our purposes the t_i values were chosen arbitrarily every 100 days in order to provide a sufficient number of points to generate the $CDF(t|M_0)$ (Figure 3). Table 2 presents the probabilities of exceeding 36,000 miles for the first 1000 days with 100-day increments. The criteria for sufficient data points were based on the convergence of the resulting distribution. The best-fit PDF based on 100-day increments (36 data points) had a 98% overlap with the best-fit PDF based on the 200-day increments (22 data points), which indicated a sufficient convergence.

Performing a best-fit distribution analysis with the complete data set produced the $f(t|36,000)$ as a lognormal distribution with parameters $\mu = 7.34$, $\sigma = 0.675$, which corresponds to a mean of 1930 days with a standard deviation of 1465 days, as presented in Figure 4.

Therefore, to approximate the cumulative percent failures, which occurred before T_0, and caused a warranty claim ($t \le T_0$, $m \le M_0$) the unreliability would have to be multiplied by the CDF of *not* exceeding M_0), as presented in equation (6):

$$F(T)_{Warranty} = F(T)_{Time-based} \int_T^{\infty} f(t|M_0)dt \quad T \le T_0 \tag{6}$$

Where $F(T)_{Warranty}$ is the percent of the total population covered by warranty for the time period T. After substituting (3) into (6) and considering the lognormal character of the mileage distribution $f(t|36,000)$, the resulting failures can be estimated by:

$$F(T)_{Warranty} = \left[1 - e^{-\left(1+\frac{\beta(T-t_S)}{t_S}\right)\left(\frac{t_S}{\eta}\right)^{\beta}}\right]\Phi\left(\frac{\ln T - \mu}{\sigma}\right) \quad t_S \le T \le T_0 \tag{7}$$

Figure 4 also shows that for this particular distribution, approximately 30% of all failures occurring within the 36 month warranty period will not be covered by warranty due to high mileage accumulation.

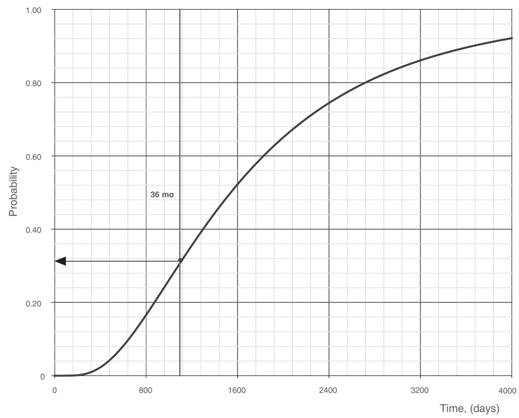

Figure 4. Probability of exceeding 36,000 miles (based on automotive dealership data).

Based on the failure probability function, the warranty cost can be calculated as:

$$W_C = n\alpha_W F(T)_{Warranty} \tag{8}$$

where:
n = production volume
α_W = average cost of repair per warranty claim.

Equation (8) along with (6) can be used to calculate the 2D cost adjustment caused by mileage accumulation and therefore reduce the expected lifecycle cost of the warranted product.

3 AUTOMOTIVE ELECTRONICS CASE STUDY

In order to illustrate the effect of 2D warranty on cost analysis, an automotive electronics example will be presented here. In this example the warranty claims numbers as well as their costs will be altered to protect the proprietary data of Delphi Corporation. The objective of this example is to forecast the IPTV for a 3-year/36,000 miles warranty on a new automotive radio with CD player with a production volume of 500,000/year and a repair cost distributed lognormally with the average of \$240 per repair. The analysis of previous models of this product produced the following failure forecasting parameters for equation (3): $t_S = 310$ days, $\beta = 0.72$, $\eta = 250{,}000$ days.

Table 3. Prediction vs. actual data (cumulative IPTV).

Days in the field	120	240	360	480	600	420	720	840	960	1080
1-D	1.48	2.01	2.53	3.05	3.58	2.79	4.10	4.62	5.15	5.67
2-D	1.48	2.00	2.49	2.93	3.29	2.72	3.57	3.77	3.90	3.97
Actual	1.32	2.04	2.50	2.90	3.21	2.71	3.44	3.61	3.79	3.91

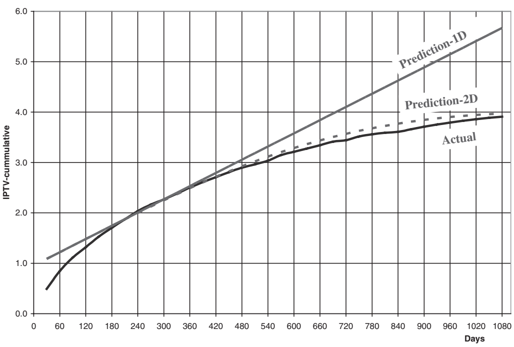

Figure 5. Cumulative incidents per thousand vehicles (IPTV) without mileage accumulation (1D), with mileage accumulation (2D), and Actual.

Table 3 presents the partial failure data obtained from the automotive dealerships in IPTV format. The complete data set is used to generate the diagram Figure 5, which shows that compared to the actual returns, the 2D-based prediction is more accurate than a 1D prediction.

As determined in the previous section the conditional probability $f(t|36,000)$ is best represented by the lognormal distribution with the parameters $\mu = 7.34$, $\sigma = 0.675$. Both Figure 4 and Figure 5 show the difference between 1D and 2D predictions as being close to 30%, which based on (8) would translate in warranty cost difference:

$$\Delta W_C = \alpha_W n \frac{IPTV_{1D} - IPTV_{2D}}{1000} \tag{9}$$

Substitution the original numbers from this case study and Table 3 into (9) would produce the cost difference of $204,000 per year, which is a significant reduction in the lifecycle cost estimate.

4 OBSERVATIONS AND CONCLUSIONS

It has been the authors' experience that the analysis of real automotive warranty claims often presents certain data processing challenges. For example, many dealership warranty claim records (especially those associated with manually entered data) often contain the data noise, such as incorrect vehicle mileages and inaccurate repair and claim dates. It is also not uncommon to have missing records of the dates and mileages. To improve the accuracy of processing, the data should be screened whenever it is possible by running some simple filters, such as positive difference between sales date and claim date or searches for missing cells.

It should also be noted that not all dealership records containing dates and mileages should be used for determining the vehicle mileage distributions. It is important to note that most of the warranty records reflect vehicles with failed parts and therefore those mileages can be biased. For example, vibration-related problems can be un-proportionally exhibited in the vehicles with high daily mileages.

Therefore the analysts calculating the mileage distributions must not use single failure mode warranty claims data, but should diversify instead. For that purpose, the best data may come from the scheduled maintenance visits, which would most likely have randomly distributed daily mileages. It is also important to analyze the different vehicle groups separately, since cars will have different mileage distributions than light trucks and heavy duty trucks will be different than off-road machinery. Also it is important to keep in mind that driving distances in Europe are different than those in the USA.

In summary, the proposed method will help to generate more accurate warranty cost estimates in the cases of 2D warranty terms. It would help to create more accurate lifecycle cost models and improve the competitiveness of the pricing strategy of products sold with warranty.

REFERENCES

Blischke, W. & Murthy D.N.P. 1994. *Warranty Cost Analysis*. New York: Marcel Dekker.

Chukova, S. & Robinson, J. 2004. Estimating Mean Cumulative Functions from Truncated Automotive Warranty. *Proceedings of the Fourth International Conference on Mathematical Methods in Reliability.* Santa Fe, New Mexico, June 21–25.

Ebeling, C. 1997. *Reliability and Maintainability Engineering*. New York: McGraw-Hill.

Iskandar, B. & Murthy, D.N.P. 2003. Repair-Replace Strategies for Two-Dimensional Warranty Policies. Mathematics and Computer Modeling. V. 38, 1233–1241.

Iskandar, B., Murthy, D.N.P. & Jack, N. 2005. A new repair-replace strategy for items sold with a two-dimensional warranty. *Computers and Operations Research*. V.32, 669–682.

Kleyner A. & Sandborn P. 2005. A warranty forecasting model based on piecewise statistical distribution and stochastic simulation. *Reliability Engineering and System Safety* 88(3): 207–214.

Krivtsov, V. & Frankstein, M. 2004. Nonparaetric Estimation of Marginal Failure Distributions from Dually Cencored Automotive Data. *Proceedings of Annual Reliability and Maintainability Symposium*, Los Angeles, CA.

Krivtsov, V. & Frankstein, M. 2006, Automotive Component Reliability: Should it be Measured in Time or Mileage? *Proceedings of Annual Reliability and Maintainability Symposium*, Newport Beach, CA.

Murthy, D.N.P. & Djamaludin, I. 2002. New product warranty: A literature review. *International Journal of Production Research* 79: 231–260.

O'Connor, P. 2003. *Practical Reliability Engineering*. Fourth Edition, NewYork: Wiley.

Rai, B. & Singh, N. 2005. A modeling framework for assessing the impact of new time/mileage warranty limits on the number and cost of automotive warranty claims. Reliability Engineering & System Safety. V. 88, 157–169

Warranty Week Newsletter 2004a. March 30, [online]. Available from: http://www.warrantyweek.com/archive/ww20040330.html, [Accessed 30 February 2006].

Warranty Week Newsletter 2004b. September 8, [online]. Available from: http://www.warrantyweek.com/archive/ww20040908.html [Accessed 20 February 2006].

Yang, G. & Zaghati, Z. 2002. Two-dimensional reliability modeling from warranty data. *Proceedings of Annual Reliability and Maintainability Symposium*. 272–278, Seattle, WA.